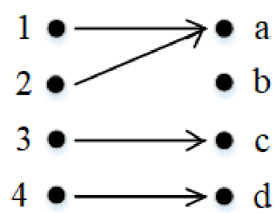
Question 77237

1 Determine whether each set is a function from $X = \{1,2,3,4\}$ to $Y = \{a,b,c,d\}$. If it is a function, find its domain and range, draw its arrow diagram, and determine if it is one-to-one, onto or both. a) $\{(1,a,),(2,a),(3,c),(4,b)\}$ b) $\{(1,c),(2,a),(3,b),(4,c),(2,d)\}$ c) $\{(1,d),(2,d),(4,a)\}$

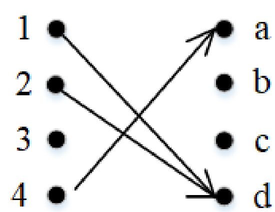
Solution

a) f is function (each $x \in X$ has the only image $y \in Y$), not one-to-one (because $f(1) = f(2) = a$), not onto (because for $d \in Y$ there is not such $x \in X$ that $f(x) = d$). Domain – X, range – $\{a,b,c\}$.



b) f is not function (because $f(2) = a$ and $f(2) = d$, i.e. for one $x \in X$ exist two different $y \in Y$).

c) f is function (each $x \in X$ has the only image $y \in Y$), not one-to-one (because $f(1) = f(2) = d$), not onto (because for $b, c \in Y$ there is not such $x \in X$ that $f(x) = b$ and $f(x) = c$). Domain – $\{1,2,4\}$, range – $\{a,d\}$.



2 List all possible functions from A to B, $A = \{a,b,c\}$, $B = \{0,1\}$. Also indicate in each case whether the function is one-to-one, is onto and one-to-one-onto.

Solution

1) $f(a) = f(b) = f(c) = 0$ not one-to-one (all $x \in A$ have the same image), not onto (because for $1 \in B$ there is not such $x \in A$ that $f(x) = 1$);

2) $f(a) = f(b) = f(c) = 1$ not one-to-one (all $x \in A$ have the same image), not onto (because for $0 \in B$ there is not such $x \in A$ that $f(x) = 0$);

3) $f(a) = f(b) = 0, f(c) = 1$ not one-to-one (for different $a, b \in A$, $\quad f(a) = f(b)$), onto (because for all $y \in B$ there exists such $x \in A$ that $f(x) = y$);

4) $f(a) = f(c) = 0, f(b) = 1$ not one-to-one (for different $a, c \in A$, $\quad f(a) = f(c)$), onto (because for all $y \in B$ there exists such $x \in A$ that $f(x) = y$);

5) $f(b) = f(c) = 0, f(a) = 1$ not one-to-one (for different $c, b \in A$, $\quad f(c) = f(b)$), onto (because for all $y \in B$ there exists such $x \in A$ that $f(x) = y$);

6) $f(a) = f(b) = 1, f(c) = 0$ not one-to-one (for different $a, b \in A$, $\quad f(a) = f(b)$), onto (because for all $y \in B$ there exists such $x \in A$ that $f(x) = y$);

7) $f(a) = f(c) = 1, f(b) = 0$ not one-to-one (for different $a, c \in A$, $\quad f(a) = f(c)$), onto (because for all $y \in B$ there exists such $x \in A$ that $f(x) = y$);

8) $f(c) = f(b) = 1, f(a) = 0$ not one-to-one (for different $ca, b \in A$, $\quad f(c) = f(b)$), onto (because for all $y \in B$ there exists such $x \in A$ that $f(x) = y$);