Answer on Question #77142 – Math – Statistics and Probability
Question

1. The Glider uses three parts of component A and two parts of component B, the Blimp uses two parts of components B and C, and the Pilot uses one part of each component. A sample of 75 components, 25 of each type, will be used to make prototypes for the various designs. The sample was analyzed and 20 of component A are proficient, 25 of component B were proficient and 15 of component C are proficient. If 30 components are selected at random, what is the likelihood two prototypes of each design can be made?

Solution

The total number of different ways to get 30 components from three types of components (consider that the number of each component $= 25 < 30$).
If we have 30 components of each type, then the total number of different ways is C_3^{30} (the number of combinations with repetition of 3 components to make 30 components sample).
Since we have 25 components of each type, we have to subtract from C_3^{30} the number of different ways which contain more than 25 components of the same type. For each type of components this number is

$$1 + 2 + C_2^2 + C_2^3 + C_2^4$$

For three types of components:

$$3 \cdot (1 + 2 + C_2^2 + C_2^3 + C_2^4)$$

Then the total number of different ways to get 30 components in our case:

$$N = \mathrm{C}_3^{30} - 3 \cdot (1 + 2 + C_2^2 + C_2^3 + C_2^4) = C_{30+3-1}^{30} - 3 \cdot (3 + C_3^2 + C_4^3 + C_5^4) =$$

$$= \frac{32!}{30!\,2!} - 3 \cdot (3 + 3 + 4 + 5) = 31 \cdot 16 - 3 \cdot 15 = 451$$

We need for two prototypes of each design: for Glider – 6 components A, 4 components B; for Blimp – 4 components B, 4 components C; for Pilot – 2 components A, 2 components B, 2 components C.
So we need:

$$component\ A: 6 + 2 = 8\ parts$$

$$component\ B: 4 + 4 + 2 = 10\ parts$$

$$component\ C: 4 + 2 = 6\ parts$$

The number of different ways such that 8 proficient components A, 10 proficient components B and 6 proficient components C are in the sample of 30 components (this is the number of combinations with repetition for remaining $30 - 8 - 10 - 6 = 6$ terms in the sample of 30 components):

$$N_1 = C_3^6 = C_{6+3-1}^6 = C_8^6 = \frac{8!}{2!\,6!} = 28$$

Answer:

The likelihood two prototypes of each design can be made:

$$p = \frac{N_1}{N} = \frac{28}{451}$$

Question

2. How to construct the joint density function?

Solution

If you have density functions $f_X(x)$ and $f_Y(y)$, then the joint density function is

$$f_{X,Y}(x, y) = f_X(x) f_Y(y)$$

Answer provided by https://www.AssignmentExpert.com