

Question

Find an expression for the trigonometric equation:

$F = 6 \sin 8\pi t - 8 \cos 8\pi t$ in the form : $R \sin(\omega t + \alpha)$ and investigate its waveform over one cycle.

Solution

Let

$$\varphi = \sin^{-1}\left(\frac{b}{\sqrt{a^2 + b^2}}\right) = \cos^{-1}\left(\frac{a}{\sqrt{a^2 + b^2}}\right) = \tan^{-1}\left(\frac{b}{a}\right)$$

then

$$\begin{aligned} a \cdot \sin(\theta) \pm b \cdot \cos(\theta) &= \sqrt{a^2 + b^2} \left(\frac{a}{\sqrt{a^2 + b^2}} \sin(\theta) \pm \frac{b}{\sqrt{a^2 + b^2}} \cos(\theta) \right) \\ &= \sqrt{a^2 + b^2} (\sin(\theta) \cdot \cos(\varphi) \pm \cos(\theta) \cdot \sin(\varphi)) = \sqrt{a^2 + b^2} \sin(\theta \pm \varphi), \end{aligned}$$

and

$$F(t) = R \cdot \sin(\omega t + \alpha) = 10 \cdot \sin(8\pi t - \sin^{-1}(0.8)),$$

where

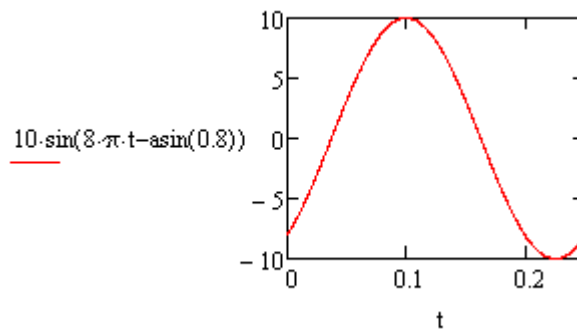
$$R = \sqrt{6^2 + 8^2} = 10 - \text{amplitude}$$

$$\omega = 8 \cdot \pi = 2 \cdot \pi \cdot f - \text{angular frequency} \rightarrow f = 4 \text{ sec}^{-1} - \text{frequency}$$

$$T = \frac{1}{f} = 0.25 \text{ sec} - \text{period}$$

$$\alpha = \sin^{-1}\left(\frac{8}{10}\right) - \text{phase shift} \rightarrow F(0) = 10 \cdot \sin\left(0 - \sin^{-1}\left(\frac{8}{10}\right)\right) = -8$$

Vertical shift is zero:



Answer: $F(t) = R \cdot \sin(\omega t + \alpha) = 10 \cdot \sin(8\pi t - \sin^{-1}(0.8))$