Given a three-dimensional surface defined implicitly by $F(x, y, z) = 0$,

$$\hat{\mathbf{n}} = \frac{\nabla F}{\sqrt{F_x^2 + F_y^2 + F_z^2}}. \qquad (*)$$

In our case F(x,y,z)= (x – 2)^2 + (y + 1)^2 + z^2 -9=0

Let’s find gradient of F:

$$\nabla F = \{F_x, F_y, F_z\} = \{\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z}\}$$

Let’s find partial derivatives:

$$\frac{\partial F}{\partial x} = \frac{\partial((x - 2)^2 + (y + 1)^2 + z^2 - 9)}{\partial x} = 2(x - 2)$$

$$\frac{\partial F}{\partial y} = \frac{\partial((x - 2)^2 + (y + 1)^2 + z^2 - 9)}{\partial y} = 2(y + 1)$$

$$\frac{\partial F}{\partial z} = \frac{\partial((x - 2)^2 + (y + 1)^2 + z^2 - 9)}{\partial z} = 2z$$

Now let’s find denominator in the formula for normal vector(*):

$$\sqrt{F_x^2 + F_y^2 + F_z^2} = \sqrt{4(x-2)^2 + 4(y+1)^2 + 4z^2} = 2\sqrt{(x-2)^2 + (y+1)^2 + z^2}$$

So we can now write down the unit normal vector:

$$\hat{n} = \frac{1}{2\sqrt{(x-2)^2 + (y+1)^2 + z^2}}\{2(x-2); 2(y+1); 2z\}$$
$$= \frac{1}{\sqrt{(x-2)^2 + (y+1)^2 + z^2}}\{(x-2); (y+1); z\}$$

Due to the statement of the question we should find normal vector at the particular point (2, 1, 4). So let’s substitute its coordinates into the formula we obtained for normal vector:

$$\hat{n}(2,1,4) = \frac{1}{\sqrt{(2-2)^2 + (1+1)^2 + 4^2}}\{(2-2); (1+1); 4\} = \frac{1}{\sqrt{4+4^2}}\{0; 2; 4\} = \frac{1}{2\sqrt{5}}\{0; 2; 4\}$$