Answer on Question #76495 – Math – Trigonometry

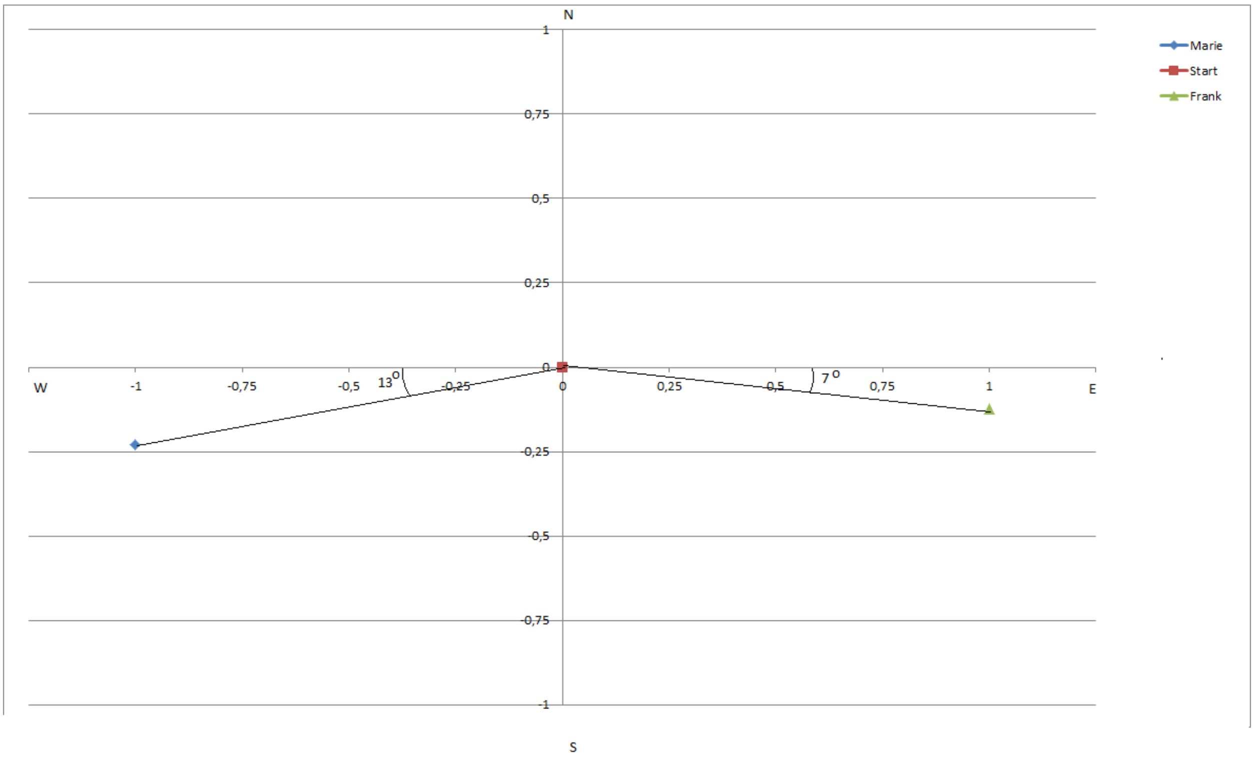
Question

Frank and Marie set sail from the same point. Frank is sailing in the direction S7∘E. Marie is sailing in the direction S13∘W. After 5 hours, Marie was 16 miles due west of Frank. How far had Marie sailed?

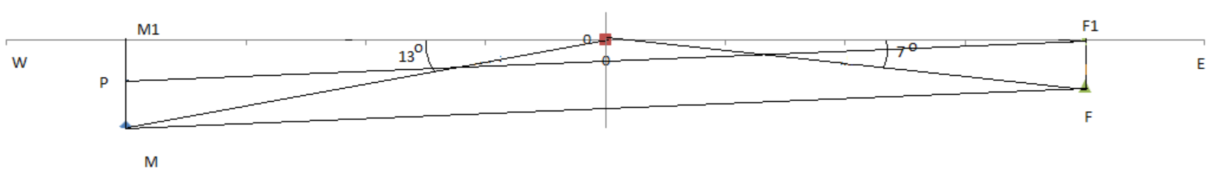
Round your answer to four decimal places.

Solution

1. Frank is sailing in the direction South-East. Marie is sailing in the direction South-West.



2. We will execute the drawing. Denote the final point of Marie by the letter M. Denote the final point of Frank by the letter F. From the points M and F we draw perpendiculars on the axis WE (points M1 and F1). Through point F1 we draw a segment parallel to MF and denote the point P.



3. The required segment in the figure OM is denoted by x (How far had Marie sailed). OF is denoted by y. M1F1=16 miles. OM1 is denoted by a, then OF1 is 16-a. If FF1 is denoted by n, then MP also n, PM1 is unknown and equal m.

4. In the triangle OMM1, $sin\angle M1OM = \frac{MM1}{OM}; sin13° = \frac{m+n}{x};$

5. In the triangle OFF1, $sin\angle F1OF = \frac{FF1}{OF}; sin7° = \frac{n}{y};$

6. In the triangle OFM, by the cosine theorem

$$MF^2 = OM^2 + OF^2 - 2 * OM * OF\ cos\angle MOF;$$

$$\angle MOF = 180° - 13° - 7° = 160°;$$

$$MF^2 = x^2 + y^2 - 2xy\ cos\angle MOF.$$

7. In the triangle M1PF1 by the Pythagorean theorem

$$PF1^2 = M1F1^2 + MM1^2;$$

$$PF1^2 = 16^2 + m^2$$

$PF1 = MF$ since we built a parallelogram, then $PF1^2 = MF^2 = 16^2 + m^2$

8. Let us formulate the system of equations.

$$\begin{cases} 16 - x\cos 13° = y\ cos7°; \\ sin13° = \dfrac{m+a}{x}; \\ sin7° = \dfrac{a}{y}; \\ 16^2 + m^2 = x^2 + y^2 - 2xy\ cos160°. \end{cases}$$

9. Calculate the values by means of the calculator:

$cos13° = 0.97437;\ cos7° = 0.99255;\ sin13° = 0.22495;\ sin7° = 0.12187;$ $cos160° = -0.93969.$

$$\begin{cases} 16 - 0.9744x = 0.99255y\ ; \\ m + n = 0.22495x; \\ n = 0.12187y; \\ 256 + m^2 = x^2 + y^2 + 1.87938xy. \end{cases}$$

10. From the second equation we substitute *n* into the third equation and express *m.* From the first equation we find *y.* We substitute all expressions into the fourth equation.

$$\begin{cases} n = 0.12187y; \ ; \\ m = 0.22495\left(\dfrac{16 - 0.97437x}{0.99255}\right) - 0.12187y; \\ y = \dfrac{16 - 0.97437x}{0.99255}; \\ 256 + (-0.12272)^2(16 - 0.97437x)^2 = x^2 + \left(\dfrac{16 - 0.97437x}{0.99255}\right)^2 + 1.89349x(16 - 0.97437x). \end{cases}$$

The last equation can be transformed and solved with respect to x.

11.

$$256 + 3.85792 - 0.46988x + 0.01431x^2 - x^2 - 257.92151 + 31.41387x - 0.95653x^2 - 30.29578x + 1.84496x^2 = 0;$$

$$-0.09726x^2 + 0.64821x + 1.93641 = 0;$$

$$D = (0.64812)^2 + 4 * 0.09726 * 1.93641 = 1.17340;$$

$$x_{1,2} = \frac{-0.64812 \pm \sqrt{1.1734}}{-2 * 0.09726} = \frac{0.64812 \mp 1.0832}{0.19452};$$

$$x_1 = 8.9007; \ x_2 = -2.2367.$$

The correct value is $x > 0$ because it is a distance, then $x_1 = 8.9007$.

Answer: 8.9007 miles.