Answer on Question #76408 – Math – Calculus

Question

1. **(a)** Find the length of the curve given by $x = t^3, y = 2t^2$ in $0 \le t \le 1$. **(b)** What is the slope of the curve at $= \frac{1}{2}$?

Solution

(a) For this problem, we will use this formula:

$$\int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$$

$$\frac{dx}{dt} = \frac{d}{dt}(t^3) = 3t^2$$

$$\frac{dy}{dt} = \frac{d}{dt}(2t^2) = 4t$$

$$\int_0^1 \sqrt{(3t^2)^2 + (4t)^2} dt = \int_0^1 \sqrt{9t^4 + 16t^2} dt = \int_0^1 t\sqrt{9t^2 + 16} =$$

$$= \begin{bmatrix} u = 9t^2 + 16 \\ du = 18tdt \to dt = \dfrac{du}{18t} \\ t = 0,..,1 \to u = 16,..,25 \end{bmatrix} = \int_{16}^{25} \frac{t\sqrt{u}}{18t} du = \frac{1}{18}\int_{16}^{25} \sqrt{u} du = \frac{1}{18}\frac{2}{3}(u^{\frac{3}{2}})|_{16}^{25} =$$

$$= \frac{1}{18}\frac{2}{3}(9t^2 + 16)^{\frac{3}{2}}|_0^1 = \frac{1}{18}\frac{2}{3}(9 * 1^2 + 16)^{\frac{3}{2}} - \frac{1}{18}\frac{2}{3}(9 * 0^2 + 16)^{\frac{3}{2}} = \frac{61}{27}$$

(b) For this problem, we will use this formula:

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}, \quad \frac{dx}{dt} \ne 0$$

$$\frac{dy}{dx} = \frac{4t}{3t^2} = \frac{4}{3t} = \left[t = \frac{1}{2}\right] = \frac{8}{3}$$

Answer:

(a) the length of the curve is $\frac{61}{27}$; **(b)** the slope of the curve is $\frac{8}{3}$.