Answer on Question #76406 – Math – Algebra

Question

Find the domain of the function $(x) = \sqrt{\frac{(4-x^2)}{[x]} + 2}$. Where [x] is greatest integer function

Solution

1. It is necessary that both conditions are met simultaneously. The radicand is greater than or equal to zero: $\frac{(4-x^2)}{[x]} + 2 \geq 0$. The denominator is not zero $[x] \neq 0$.

2. The integer part of x is the largest integer n not exceeding x.

$[x] = 0 \; if \; x \in [0; 1)$. Consequently $[x] \neq 0 \; if \; x \in (-\infty; 0) \cup [1; \infty)$.

3. Solve the inequality:

$$\frac{(4-x^2)}{[x]} + 2 \geq 0.$$

To do this, we first solve the system of equations (The substitution $[x]$ by x will not affect the findings of values when the numerator of the fractions is zero.):

$$\begin{cases} \dfrac{(4-x^2)}{x} + 2 = 0; \\ [x] \neq 0. \end{cases}$$

We multiply the terms of the equation by the denominator:

$$4 - x^2 = -2x \text{ or } -x^2 + 2x + 4 = 0.$$

We solve this equation through the discriminant.

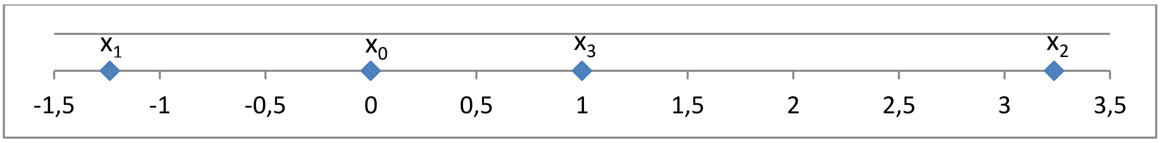
$$D = 2^2 - 4 * (-1) * 4 = 20;$$

$$x_{1,2} = \frac{-2 \pm \sqrt{20}}{2*(-1)} = \frac{2 \mp 2\sqrt{5}}{2} = 1 \mp \sqrt{5};$$

$$x_1 = 1 - \sqrt{5}; x_2 = 1 + \sqrt{5}.$$

We build a numerical line and apply points.

$$x_0 = 0;\; x_1 = 1 - \sqrt{5} \approx -1.236; x_2 = 1 + \sqrt{5} \approx 3.236;\; x_3 = 1.$$



We obtained five numerical intervals. We define the sign of the expression $\frac{(4-x^2)}{[x]} + 2$ on each of them.

1). $(-\infty; 1 - \sqrt{5}]$: if $x = -3$, then $\frac{4-(-3)^2}{[-3]} + 2 = \frac{4-9}{-3} + 2 = \frac{5}{3} + 2 > 0$;

2). $(1 - \sqrt{5}; 0)$: if $x = -1$, then $\frac{4-(-1)^2}{[-1]} + 2 = \frac{4-1}{-1} + 2 = -3 + 2 < 0$;

3). $[0;1)$: the function $f(x)$ does not exist on this interval.

4). $[1;1+\sqrt{5}]$: if $x=3$, then $\frac{4-3^2}{[3]}+\ 2=\frac{4-9}{3}+2=-\frac{5}{3}+2>0$;

5). $[1+\sqrt{5};+\infty)$: if $x=5$, then $\frac{4-5^2}{[5]}+\ 2=\frac{4-25}{5}+2=\frac{-21}{5}+2<0$.

We write out all the intervals where the function is defined and the radicand is nonnegative.

$$x\in(-\infty;1-\sqrt{5}]\cup[1;1+\sqrt{5}].$$

Answer: the domain of the function $f(x)\ =\sqrt{\frac{(4-x^2)}{[x]}+\ 2}$ is $x\in(-\infty;1-\sqrt{5}]\cup[1;1+\sqrt{5}]$.