Answer on Question #76403 – Math – Algebra

Question

Check whether the function f, defined by $f(x) = \cos x - \cos 3x$, is periodic or not.

Solution

1. The function $f(x)$ is periodic, If $f(x+T) = f(x-T) = f(x), T \neq 0$, T is the period.

2. We transform expression:

$$cosx - cos3x = -2sin\frac{x+3x}{2}sin\frac{x-3x}{2} = -2sin2x * \sin(-x) = 2sinx\ sin2x;$$

The product of periodic functions is also a periodic function.

The period of $\sin x$ is $T_1 = 2\pi$; the period of $\sin 2x$ is $T_2 = \pi$.

The least common multiple$T_1 and\ T_2\ \ is\ 2\pi.$

$$T = lcm(T_1, T_2) = lcm(\pi, 2\pi) = 2\pi.$$

3. Perform the test:

$$\begin{aligned} f(x+2\pi) = \ &\cos(x+2\pi) - \cos 3(x+2\pi) = \\ &= \cos\ x\cos 2\pi - \ \sin x \sin 2\pi + \cos\ 3x \cos 6\pi - \ \sin 3x \sin 6\pi = \\ &= \cos\ x * 1 - \ \sin x * 0 - \cos\ 3x * 1 + \ \sin 3x * 0 = \cos x - \cos 3x = f(x) \end{aligned}$$

$$\begin{aligned} f(x-2\pi) = \ &\cos(x+2\pi) - \cos 3(x+2\pi) = \\ &= \cos\ x\cos 2\pi + \ \sin x \sin 2\pi - \cos\ 3x \cos 6\pi - \ \sin 3x \sin 6\pi = \\ &= \cos\ x * 1 + \ \sin x * 0 - \cos\ 3x * 1 - \ \sin 3x * 0 = \cos x - \cos 3x = f(x) \end{aligned}$$

The periodicity condition for the function is satisfied:

$$f(x+T) = f(x-T) = f(x),\ T \neq 0.$$

Answer: The function $f(x) = \cos x - \cos 3x$ is periodic, $T = 2\pi$ is the period.