Answer on Question #76401 – Math – Algebra

Question

Trace the curve: $y^2(x+1) = x^2(3-x)$, clearly stating all the properties used for tracing it.

Solution

1. Symmetry

The curve is not symmetrical about the y −axis.

The curve is symmetrical about the x −axis.

$$(-y)^2(x+1) = x^2(3-x) => y^2(x+1) = x^2(3-x)$$

The curve is not symmetrical in opposite quadrants.

The curve is not symmetrical about the line $y = x$.

2. Origin.

The curve passes through the origin

$$x = 0 => y = 0$$

The equations of the tangents to the curve at the origin is obtained by equating the lowest degree terms in x and y in the given equation to zero

$$y^2(x+1) = x^2(3-x)$$

$$xy^2 + y^2 = 3x^2 - x^3$$

$$y^2 = 3x^2 => y = \pm\sqrt{3}x$$

First tangent: $y = \sqrt{3}x$.

Second tangent: $y = -\sqrt{3}x$.

The tangents are real and distinct.

The origin is node.

3. Intersection with the coordinate axes.

$y - intercept: x = 0 => y = 0, point(0,0)$

$x - intercept: y = 0 => 0 = 3x^2 - x^3$

$x^2(3-x) = 0$

$x = 0 \quad or \quad x = 3$

$point(0,0), point(3,0)$

4. First derivative

Take derivative with respect to x of both sides of the equation and use the Chain rule

$$\frac{d}{dx}(xy^2 + y^2) = \frac{d}{dx}(3x^2 - x^3)$$

$$y^2 + 2xy\frac{dy}{dx} + 2y\frac{dy}{dx} = 6x - 3x^2$$

$$\frac{dy}{dx} = \frac{6x - 3x^2 - y^2}{2xy + 2y}$$

We have the vertical tangent $x = 3$ at the $point(3, 0)$.

5. Asymptote(s)
$xy^2 + y^2 = 3x^2 - x^3$
Asymptote parallel to y −axis
$x + 1 = 0 => x = -1$
Vertical asymptote: $x = -1$.
There is no horizontal asymptote.
There is no slant (oblique) asymptote.

6. Regions where no part of the curve lies.
$y^2(x + 1) = x^2(3 - x)$
$$y = \pm\sqrt{\frac{x^2(3 - x)}{x + 1}}$$
$$\frac{x^2(3 - x)}{x + 1} \geq 0 => -1 < x \leq 3$$

7. Increasing and decreasing
$$\frac{dy}{dx} = \frac{6x - 3x^2 - y^2}{2xy + 2y}, -1 < x \leq 3$$
$$\frac{dy}{dx} = 0 => 6x - 3x^2 - y^2 = 0, x \neq 0, -1 < x \leq 3$$
$$6x - 3x^2 - \frac{x^2(3 - x)}{x + 1} = 0$$
$6x^2 + 6x - 3x^3 - 3x^2 - 3x^2 + x^3 = 0$
$-2x^3 + 6x = 0$
$x(3 - x^2) = 0$
We take $x = \sqrt{3}$
$$y|_{x=\sqrt{3}} = \pm\sqrt{\frac{3(3 - \sqrt{3})}{\sqrt{3} + 1}}$$
$$y_1 = \sqrt{\frac{x^2(3 - x)}{x + 1}}$$
$If -1 < x < \sqrt{3}, y_1\ decreases\ from\ \infty$
$If\ \sqrt{3} < x < 3, y_1\ increases$

$$y_2 = -\sqrt{\frac{x^2(3 - x)}{x + 1}}$$

$If\ -1 < x < \sqrt{3}, y_2\ increases\ from -\infty$
$If\ \sqrt{3} < x < 3, y_2\ decreases$