## Answer on Question #76401 – Math – Algebra

## Question

Trace the curve:  $y^2(x + 1) = x^2(3 - x)$ , clearly stating all the properties used for tracing it.

## **Solution**

1. Symmetry

The curve is not symmetrical about the y –axis. The curve is symmetrical about the x –axis.

$$(-y)^{2}(x+1) = x^{2}(3-x) \Longrightarrow y^{2}(x+1) = x^{2}(3-x)$$

The curve is not symmetrical in opposite quadrants. The curve is not symmetrical about the line y = x.

2. Origin.

The curve passes through the origin

$$x = 0 => y = 0$$

The equations of the tangents to the curve at the origin is obtained by equating the lowest degree terms in x and y in the given equation to zero

$$y^{2}(x + 1) = x^{2}(3 - x)$$
  

$$xy^{2} + y^{2} = 3x^{2} - x^{3}$$
  

$$y^{2} = 3x^{2} = y = \pm\sqrt{3}x$$

First tangent:  $y = \sqrt{3}x$ . Second tangent:  $y = -\sqrt{3}x$ . The tangents are real and distinct. The origin is node.

3. Intersection with the coordinate axes.  $y - intercept: x = 0 \Rightarrow y = 0, point(0, 0)$   $x - intercept: y = 0 \Rightarrow 0 = 3x^2 - x^3$   $x^2(3 - x) = 0$  x = 0 or x = 3point(0, 0), point(3, 0)

4. First derivative

Take derivative with respect to x of both sides of the equation and use the Chain rule

$$\frac{d}{dx}(xy^{2} + y^{2}) = \frac{d}{dx}(3x^{2} - x^{3})$$
$$y^{2} + 2xy\frac{dy}{dx} + 2y\frac{dy}{dx} = 6x - 3x^{2}$$
$$\frac{dy}{dx} = \frac{6x - 3x^{2} - y^{2}}{2xy + 2y}$$

We have the vertical tangent x = 3 at the *point*(3, 0).

5. Asymptote(s)  $xy^2 + y^2 = 3x^2 - x^3$ Asymptote parallel to y -axis  $x + 1 = 0 \Longrightarrow x = -1$ Vertical asymptote: x = -1. There is no horizontal asymptote. There is no slant (oblique) asymptote.

6. Regions where no part of the curve lies.  $y^{2}(x + 1) = x^{2}(3 - x)$   $y = \pm \sqrt{\frac{x^{2}(3 - x)}{x + 1}}$   $\frac{x^{2}(3 - x)}{x + 1} \ge 0 \Longrightarrow -1 < x \le 3$ 

7. Increasing and decreasing  

$$\frac{dy}{dx} = \frac{6x - 3x^2 - y^2}{2xy + 2y}, -1 < x \le 3$$

$$\frac{dy}{dx} = 0 \Longrightarrow 6x - 3x^2 - y^2 = 0, x \ne 0, -1 < x \le 3$$

$$6x - 3x^2 - \frac{x^2(3 - x)}{x + 1} = 0$$

$$6x^2 + 6x - 3x^3 - 3x^2 - 3x^2 + x^3 = 0$$

$$-2x^3 + 6x = 0$$

$$x(3 - x^2) = 0$$
We take  $x = \sqrt{3}$ 

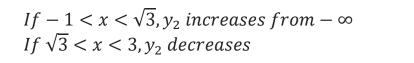
$$y|_{x=\sqrt{3}} = \pm \sqrt{\frac{3(3 - \sqrt{3})}{\sqrt{3} + 1}}$$

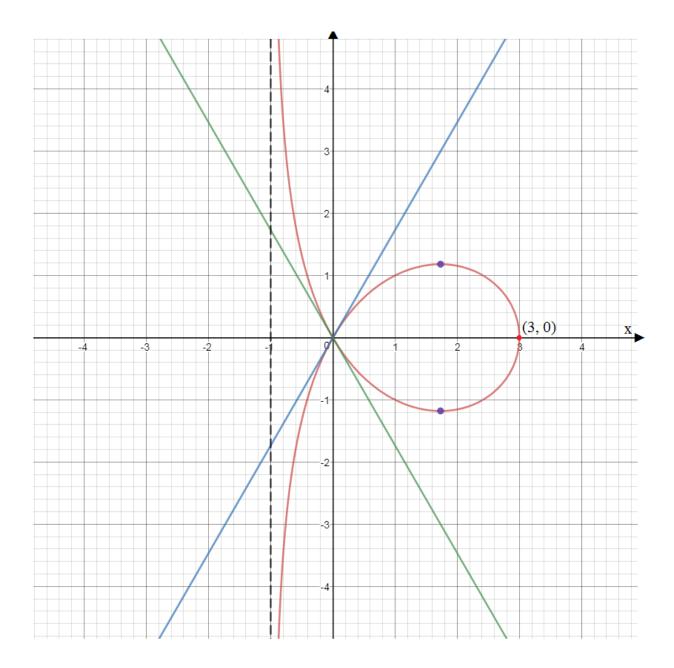
$$y_1 = \sqrt{\frac{x^2(3 - x)}{x + 1}}$$

$$If - 1 < x < \sqrt{3}, y_1 \text{ decreases from } \infty$$

$$If \sqrt{3} < x < 3, y_1 \text{ increases}$$

$$y_2 = -\sqrt{\frac{x^2(3-x)}{x+1}}$$





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