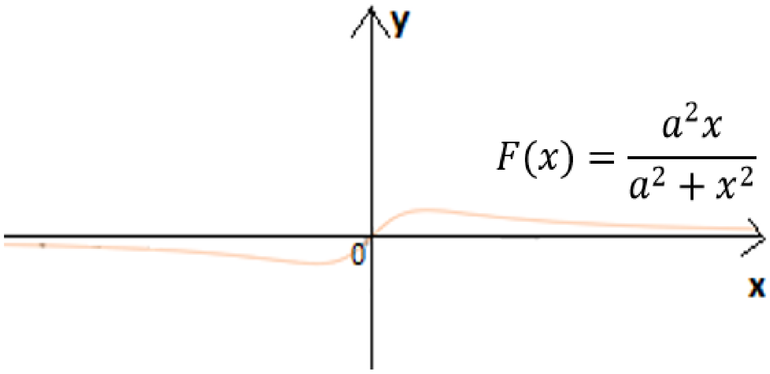
Answer on Question #76397 – Math – Calculus



Question:

Trace the curve y(a^2+ x^2) = a^2x

Solution:

We have function

$$F(x) = \frac{a^2x}{a^2 + x^2}$$

1) Domain:

$x \epsilon R$, so function does not have vertical asymptotes. If $a = 0$, then the curve is $y = 0$. If $a \neq 0$, then $x^2 + a^2 \neq 0$ and the curve will exist.

2) **Symmetry:**

Function is odd:

$$F(-x) = -F(x)$$

$$\frac{a^2x}{a^2 + x^2} = \frac{a^2(-x)}{a^2 + (-x)^2} = -\frac{a^2x}{a^2 + x^2}$$

So the curve is symmetric about the origin.

3) **Origin:** The curve passes through the origin since the equation does not contain any constant term.

4) **Intercept on y-axis:** Putting x=0 in the equation we get

$$y = \frac{a^2 \cdot 0}{a^2 + 0} = 0$$

5) **Intervals of monotonicity:**

$$F'(x) = \frac{a^2(a^2 - x^2)}{(a^2 + x^2)^2} = \frac{a^2(a - x)(a + x)}{(a^2 + x^2)^2}$$

Roots: $\begin{cases} a = 0 \\ x \neq 0 \end{cases} \begin{cases} x = a \\ a \neq 0 \end{cases} \begin{cases} x = -a \\ a \neq 0 \end{cases}$

So, function decreases when $x \epsilon (-\infty; -a) \cup (a; +\infty)$

function increases when $x\epsilon(-a; a)$

6) **Inflection points:**

$$F''(x) = \frac{2a^2x^3 - 6a^4x}{(a^2 + x^2)^3}$$

$$F''(x) = 0$$

$x = 0, x = \sqrt{3}a, x = -\sqrt{3}a$ are inflection points

7) **Slant asymptotes:**

$$y = kx + b$$

$$k = \lim_{x \to \infty} \frac{F(x)}{x}$$

If there is no limit, then there is no slant asymptotes.
If $k = 0$, then $y = b$ is a horizontal asymptote.

$$k = \lim_{x\to\infty} \frac{\frac{a^2x}{a^2+x^2}}{x} = \frac{a^2x}{x(a^2+x^2)} = \frac{a^2}{a^2+x^2} = 0,$$

$$b = \lim_{x\to\infty}(F(x) - kx) = \lim_{x\to\infty}(\frac{a^2x}{a^2+x^2}) = 0.$$

Thus, the function has the horizontal asymptote $y = 0$.