Answer on Question #76361 – Math – Discrete Mathematics

Question

Prove that $A - (B \cup C) = (A - B) \cap (A - C)$.

Proof

To prove the given equality of sets, we take an arbitrary element from the left set and show that it lies in the right set and vice versa.

Let $a \in A - (B \cup C)$. Then $a \in A$ and $a \notin (B \cup C)$. The relation $a \notin (B \cup C)$ means that $a \notin B$ and $a \notin C$ at the same time. Therefore, we have that $a \in A$ and $a \notin B$ and $\notin C$, i.e. $a \in A - B$ and $a \in A - C$. The last statement means that $a \in (A - B) \cap (A - C)$. Consequently, $A - (B \cup C) \subset (A - B) \cap (A - C)$.

Let now $a \in (A - B) \cap (A - C)$. Then $a \in A - B$ and $a \in A - C$, i.e. $a \in A$ and $a \notin B$ and $a \notin C$. The statements $a \notin B$ and $a \notin C$ means that $a \notin (B \cup C)$. In this way, we have that $a \in A$ and $a \notin (B \cup C)$. Therefore, $a \in A - (B \cup C)$.

This, in turn, gives $(A - B) \cap (A - C) \subset A - (B \cup C)$.

Finally, from inclusions $A - (B \cup C) \subset (A - B) \cap (A - C)$ and

$(A - B) \cap (A - C) \subset A - (B \cup C)$ it follows that $A - (B \cup C) = (A - B) \cap (A - C)$.