Answer on Question #76360 – Math – Discrete Mathematics

Question

Prove that for all sets A and B,

$$A \bigcup B^C = B^C \bigcup (A \bigcap B).$$

Solution

Let $x \in A \bigcup B^C$. Then $x \in A$ or $x \in B^C$.

If $x \in B^C$, then $x \in B^C \bigcup (A \bigcap B)$.

If $x \in A$ and $x \notin B^C$, then $x \in A$ and $x \in B$. So, $x \in A \bigcap B$. Then $x \in B^C \bigcup (A \bigcap B)$.

Therefore, in each case $x \in B^C \bigcup (A \bigcap B)$ and so $A \bigcup B^C \subseteq B^C \bigcup (A \bigcap B)$.

Let $x \in B^C \bigcup (A \bigcap B)$. Then $x \in B^C$ or $x \in A \bigcap B$.

If $x \in B^C$, then $x \in A \bigcup B^C$.

If $x \in A \bigcap B$, then $x \in A$ and so $x \in A \bigcup B^C$.

Therefore, in each case $x \in A \bigcup B^C$ and so $B^C \bigcup (A \bigcap B) \subseteq A \bigcup B^C$.

Since $A \bigcup B^C \subseteq B^C \bigcup (A \bigcap B)$ and $B^C \bigcup (A \bigcap B) \subseteq A \bigcup B^C$, $A \bigcup B^C = B^C \bigcup (A \bigcap B)$.

Hence,

$$A \bigcup B^C = B^C \bigcup (A \bigcap B).$$

Another way:

Let X be an universal set. Then

$$B^C \bigcup (A \bigcap B) = \left(B^C \cup A\right) \cap \left(B^C \cup B\right) = \left(B^C \cup A\right) \cap X = B^C \cup A = A \bigcup B^C.$$

Hence,

$$A \bigcup B^C = B^C \bigcup (A \bigcap B).$$