Answer on Question #76358 – Math – Discrete Mathematics

Question

Prove that for all sets A and B:
$A \subseteq B \Longleftrightarrow A \cup B = B$

Solution

1) $A \subseteq B \Rightarrow A \cup B = B$.

Assume that $A \subseteq B \Rightarrow A \cup B \neq B$. As all elements from B are contained in A∪B, there exists x, such as $\begin{cases} x \in A \cup B \\ x \notin B \end{cases}$. Consequently $\begin{cases} x \in A \\ x \notin B \end{cases}$. But using that fact and A⊆B one gets a contradiction. Thus, $A \subseteq B \Rightarrow A \cup B = B$ holds true.

2) $A \subseteq B \Leftarrow A \cup B = B$. If not, there exists an element x from A that isn't contained in B. Consequently $x \in A \cup B$ which is equal to B and it comes to contradiction.
Thus, $A \subseteq B \Leftarrow A \cup B = B$ holds true.

It follows from 1) and 2) that $A \subseteq B \Longleftrightarrow A \cup B = B$.