Answer on Question #76327 – Math – Discrete Mathematics

Question

How many ways can you assign 4 caretakers each of which look after 2 lighthouses for a year (for a total of 8 lighthouses), and then assign each two lighthouses in the next year so that none of them tend the same two lighthouses both years?

Solution

For the first year:

$$N_1 = C_8^2 \cdot C_6^2 \cdot C_4^2 = \frac{8!}{6!\,2!} \cdot \frac{6!}{4!\,2!} \cdot \frac{4!}{2!\,2!} = 28 \cdot 15 \cdot 6 = 2520$$

For the second year:

$$N_2 = (C_8^2 - 1)(C_6^2 - 1)(C_4^2 - 1) = 27 \cdot 14 \cdot 5 = 1890$$

In the second year each caretaker cannot have the same two lighthouses as in the first year, so 1 way (of the first year) for each caretaker is restricted. Thus, we have the given formula for N_2. In total, there will be $N_1 N_2$ ways.