Answer on Question #76325 – Math – Discrete Mathematics

Question

Find a generating function in closed form for the sequence: {1,2,3,4,1,2,3,4,1,2,3,4,...}.

Solution

$$1 + 2x + 3x^2 + 4x^3 + x^4 + 2x^5 + \ldots = \sum_{k=0}^{\infty}(x^{4k} + 2x^{4k+1} + 3x^{4k+2} + 4x^{4k+3}) =$$
$$\sum_{k=0}^{\infty}(1 + 2x + 3x^2 + 4x^3)x^{4k} = (1 + 2x + 3x^2 + 4x^3)\sum_{k=0}^{\infty} x^{4k} =$$
$$(1 + 2x + 3x^2 + 4x^3) \cdot \frac{1}{1-x^4} = \frac{1+2x+3x^2+4x^3}{1-x^4}.$$

Answer: $\frac{1+2x+3x^2+4x^3}{1-x^4}$.