Question

Find the indicated limit $\lim\limits_{x \to -1} f(x)$, if it exists, where

$$f(x) = \begin{cases} 4 - x, & x < -1 \\ 5, & x = -1 \\ x + 6, & x > -1 \end{cases}$$

Solution

Let us evaluate the one-sided limits of the function f at the point -1 and compare them.

$$\lim_{x \to -1-0} f(x) = \lim_{x \to -1-0} (4 - x) = 5; \quad \lim_{x \to -1+0} f(x) = \lim_{x \to -1+0} (x + 6) = 5;$$

$$\lim_{x \to -1-0} f(x) = \lim_{x \to -1+0} f(x) = 5, \text{ hence } \lim_{x \to -1} f(x) = 5.$$

Let us notice that $\lim\limits_{x \to -1} f(x) = 5 = f(-1)$, so $f \in C_{\{-1\}}$. Moreover, this function is continuous on the whole number axis.

Answer: $\lim\limits_{x \to -1} f(x) = 5$.