Answer on Question #76282 – Math – Calculus

Question

Prove the limit by epsilon - delta definition

$$\lim_{x\to 3}(x^2+4)=13.$$

Proof

$$\lim_{x\to a} f(x)=b \overset{def}{\Longleftrightarrow} \forall\, \varepsilon>0\ \exists\, \delta=\delta(\varepsilon)>0\ \ \forall x\in R\colon 0<|x-a|<\delta \Rightarrow$$

$\Rightarrow\ |f(x)-b|<\varepsilon$. We have $a=3, b=13, f(x)=x^2+4$.

Let's choose and fix any $\varepsilon>0$.

Consider $|f(x)-b|=|x^2+4-13|=|x^2-9|=|x^2-6x+9+6x-18|=$

$=|(x-3)^2+6(x-3)|\le|x-3|^2+6|x-3|=(|x-3|+3)^2-9.$

Let $(|x-3|+3)^2-9=\varepsilon_0$ then $|x-3|=\sqrt{\varepsilon_0+9}-3.$

Let $\delta(\varepsilon)=\sqrt{\varepsilon+9}-3$ then $\forall\, x\in R\colon 0<|x-3|<\delta \Rightarrow\ |f(x)-13|<\varepsilon$■