Question

Find domain and range of $f(x)$:
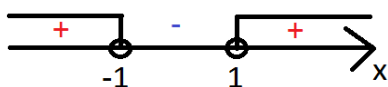
$$f(x) = \frac{1}{\sqrt{x^2 - 1}}$$

Solution

i) Domain ($D(f)$):

$$\begin{cases} \sqrt{x^2 - 1} \neq 0 \\ x^2 - 1 \geq 0 \end{cases} \Leftrightarrow x^2 - 1 > 0$$

$x^2 - 1 > 0$

$x^2 > 1$

$x = 1 \quad x = -1$

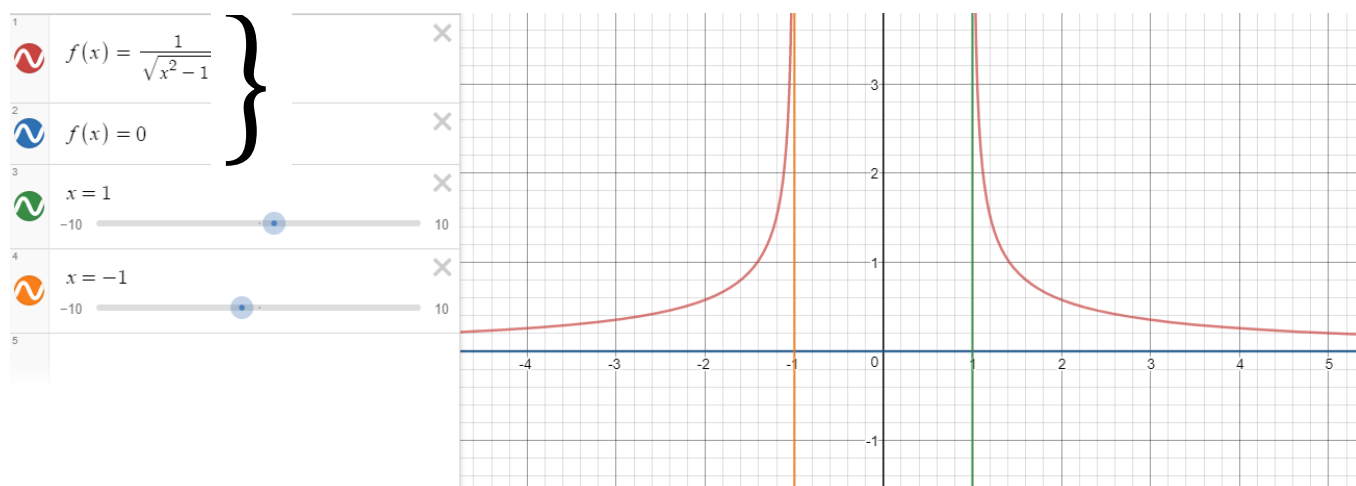$x \in (-\infty; -1) \cup (1; \infty) \Rightarrow D(f) = (-\infty; -1) \cup (1; \infty)$

ii) Range ($E(f)$):

We construct a graph of the function $f(x) = \frac{1}{\sqrt{x^2-1}}$:

$\begin{matrix} f(x) = 0 \\ x = -1 \\ x = 1 \end{matrix} \quad -$ asymptotes

$E(f) = (0; \infty)$.

Answer: $D(f) = (-\infty; -1) \cup (1; \infty)$; $E(f) = (0; \infty)$.