Answer on Question #76274 – Math – Algebra

Question

Find domain and range of

$$f(x) = \frac{1}{1-\sin(x)}$$

Solution

Domain is D(f):

$$1 \neq \sin(x) => x \neq \frac{\pi}{2} + 2\pi k, k \text{ is integer.}$$

So, D(f) = (-∞; +∞)\\{$\frac{\pi}{2} + 2\pi k$}, k is integer.

Range is E(f).

First method (easier):

$$\begin{cases} sinx \geq -1 \\ sinx < 1 \end{cases}; \begin{cases} -sinx \leq 1 \\ -sinx > -1 \end{cases}; \begin{cases} 1 - sinx \leq 2 \\ 1 - sinx > 0 \end{cases}; \begin{cases} \frac{1}{1-sinx} \geq \frac{1}{2} \\ 1 - sinx > 0 \end{cases};$$

Then we have: E(f) = $[\frac{1}{2}; +\infty)$.

Second method (harder):

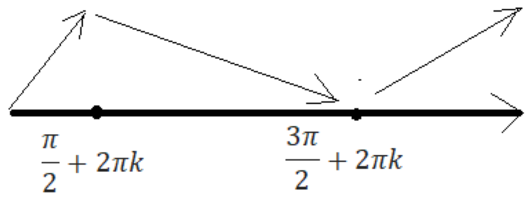
If we find min and max, we will have the range of f(x).

1) $(\frac{1}{1-\sin(x)})' = -\frac{1}{(1-sinx)^2} * (1\text{-}sinx)' = -\frac{(-cosx)}{(1-sinx)^2} = \frac{cosx}{(1-sinx)^2}$

2) $\frac{cosx}{(1-sinx)^2} > 0;\ (1-sinx)^2 > 0\ \forall x \in \mathbb{R};$

Point of extremum of f(x) is $\frac{\pi}{2} + 2\pi k$ and $\frac{3\pi}{2} + 2\pi k$

Draw the line of numbers:



So, f$(\frac{\pi}{2}+2\pi k)$ is maximum and f$(\frac{3\pi}{2}+2\pi k)$ is minimum. But , f$(\frac{\pi}{2}+2\pi k)$ does not exist, then we have minimum only.

$$f(\frac{3\pi}{2}+2\pi k)=\frac{1}{2},$$

$$E(f)=[\frac{1}{2};+\infty).$$

Answer: D(f) = $(-\infty;+\infty)\backslash\{\frac{\pi}{2}+2\pi k\}$, k is integer; E(f) = $[\frac{1}{2};+\infty)$.