Answer on Question #76273 – Math – Differential Equations

Question

If $\frac{d^2x}{dt^2} + \frac{g(x-a)}{b} = 0$, (a, b, g being positive constants) and x = a′ and $\frac{dx}{dt} = 0$ when t=0, show that x = a+ (a'-a) cos $\{\sqrt{(\frac{g}{b})}t\}$

Solution

Given, differential equation:

$$\frac{d^2x}{dt^2} + \frac{g(x-a)}{b} = 0 \text{ ....(1)}$$

Boundary conditions are given by, x = a′ and $\frac{dx}{dt} = 0$ at t=0.

Now, solution of the equation (1) is

$$x - a = A\cos\{\sqrt{(\tfrac{g}{b})}t\} + B\sin\{\sqrt{(\tfrac{g}{b})}t\} \text{ ....(2)}$$

At t= 0, x = a′ then from equation (2) we get,

$$A = a' - a \text{ ....(3)}$$

Now, take the derivative of equation (2) and we get,

$$\frac{dx}{dt} = [-A\sin\{\sqrt{(\tfrac{g}{b})}t\} + B\cos\{\sqrt{(\tfrac{g}{b})}t\}]\,\{\sqrt{(\tfrac{g}{b})}\} \text{ ....(4)}$$

At, t = 0, $\frac{dx}{dt} = 0$ then from equation (3) we get,

$$B = 0.$$

Now, put the value of A and B in equation (2) and we get,

$$x - a = (a' - a)\cos\{\sqrt{(\tfrac{g}{b})}t\}$$

or,

$$x = a + (a' - a)\cos\{\sqrt{(\tfrac{g}{b})}t\}$$

Answer: x = a + (a′-a) cos $\{\sqrt{(\frac{g}{b})}t\}$.