Answer on Question #76255 – Math – Discrete Mathematics.

Question

Let $X=\{1,2,3\}$. Define a relation $\sim$ on $P(X)$ by $A\sim B$ if A and B have the same number of elements.

Prove that $\sim$ is an equivalence relation. Write down all equivalence classes of $\sim$.

Solution

Reflexively: Let $A\in P(x)$. Then $A\sim A$ because the set has an unaltered number of elements.

Symmetric: Let $A\in P(x)$, $B\in P(x)$ and $A\sim B$. Since $A\sim B$, $|A|=|B|$. Then $|B|=|A|$. Since $|B|=|A|$, $B\sim A$.

Transitivity: Let $A\in P(x)$, $B\in P(x)$, $C\in P(x)$, $A\sim B$ and $B\sim C$. Since $A\sim B$, $|A|=|B|$. Since $B\sim C$, $|B|=|C|$. Then $|A|=|C|$ because $|A|=|B|$ and $|B|=|C|$. Since $|A|=|C|$, $A\sim C$.

Equivalence classes of $\sim$:

$[\varnothing]=\{\varnothing\}$ (zero of elements),

$[\{1\}]=\{\{1\},\{2\},\{3\}\}$ (one element),

$[\{1,2\}]=\{\{1,2\},\{1,3\},\{2,3\}\}$ (two elements),

$[\{1,2,3\}]=\{\{1,2,3\}\}$ (three elements).