

Answer on Question #76240 – Math – Discrete Mathematics

Question

Let X be a non-empty set, and let R be an equivalence relation on X . Let C be the set of all equivalence classes of R . So $C = \{A \subseteq X : \text{such that } A = [x] \text{ for some } x \in X\}$.

Now, define $f : X \rightarrow C$ by the rule $f(x) = [x]$ for all $x \in X$.

Suppose $X = \{1, 2, 3, 4, 5\}$ and that R is an equivalence relation for which $1R3$, $2R4$ but $1 \not R 2$, $1 \not R 5$, and $2 \not R 5$.

Write down the equivalence classes of R and draw a diagram to represent the function f .

Solution

Since $1R3$, $[1] = \{1, 3\}$. Since $2R4$, $[2] = \{2, 4\}$. Note that $[1] \neq [2]$ because $1 \not R 2$. Also $5 \notin [1]$ and $5 \notin [2]$ because $1 \not R 5$, and $2 \not R 5$. Since R is reflexive, $5R5$. Thus $5 \in [5]$. Therefore there are three equivalence classes: $[1]$, $[2]$ and $[5]$.

Checking: $[1] \cup [2] \cup [5] = \{1, 3\} \cup \{2, 4\} \cup \{5\} = \{1, 2, 3, 4, 5\} = X$.

