Answer on Question #76239 – Math – Discrete Mathematics

Question

Let X be a non-empty set, and let R be an equivalence relation on X. Let C be the set of all equivalence classes of R. So $C = \{A \subseteq X : \text{such that } A = [x] \text{ for some } x \in X\}$.

Now, define $f : X \to C$ by the rule $f(x) = [x]$ for all $x \in X$. Prove that if $x \in X$, then there is one and only one equivalence class which contains x.

Solution

Since R is an equivalence relation, R is reflexive. Then xRx for each $x \in X$. Then for each $x \in X$ there is the equivalence classes $[x]$ such that $x \in [x]$.

Suppose that $x \in [a]$ and $x \in [b]$ with not aRb. Since $x \in [a]$ and $x \in [b]$, xRa and xRb. By symmetry of R it follows that aRx. By transitivity of R we have that aRb (aRx and xRb). We got a contradiction with the assumption.

Hence, for all $x \in X$ there is one and only one equivalence class which contains x.