Answer on Question #76238 – Math – Discrete Mathematics.

Question

Let $X = \{1, 2, 3\}$. Define a relation $\sim$ on $P(X)$ by $A \sim B$ if A and B have the same number of elements.

Prove that $\sim$ is an equivalence relation. Write down all equivalence classes of $\sim$.

Solution

<u>Reflexively:</u> Let $A \in P(x)$. Then $A \sim A$ because the set has an unaltered number of elements.

<u>Symmetric:</u> Let $A \in P(x)$, $B \in P(x)$ and $A \sim B$. Since $A \sim B$, $|A| = |B|$. Then $|B| = |A|$. Since $|B| = |A|$, $B \sim A$.

<u>Transitivity:</u> Let $A \in P(x)$, $B \in P(x)$, $C \in P(x)$, $A \sim B$ and $B \sim C$. Since $A \sim B$, $|A| = |B|$. Since $B \sim C$, $|B| = |C|$. Then $|A| = |C|$ because $|A| = |B|$ and $|B| = |C|$. Since $|A| = |C|$, $A \sim C$.

Equivalence classes of $\sim$:

$[\varnothing] = \{\varnothing\}$ (zero of elements),

$[\{1\}] = \{\{1\}, \{2\}, \{3\}\}$ (one element),

$[\{1, 2\}] = \{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$ (two elements),

$[\{1, 2, 3\}] = \{\{1, 2, 3\}\}$ (three elements).