Answer on Question #76224 – Math – Discrete Mathematics

Question

Prove or give a counterexample to the following: For a set A and binary relation R on A, if R is reflexive and symmetric, then R must be transitive as well.

Solution

This statement is not true.

Consider the set $A=\{a,b,c\}$ and the binary relation

$$R=\{(a,a),(b,b),(c,c),(a,b),(b,a),(b,c),(c,b)\}$$

on A. Then R is reflexive because $(a,a),(b,b),(c,c)\in R$, i.e. $(x,x)\in R$ for all $x\in A$. This relation is symmetric because if $(x,y)\in R$ then $(y,x)\in R$ for each $x,y\in A$. But R is not transitive: $(a,b)\in R$ and $(b,c)\in R$ but $(a,c)\notin R$.

Answer: R is not transitive.