# ANSWER on Question #76193 - Math - Discrete Mathematics

Find domain and range of (answers should be subsets of $\mathbb{R}$ ):

$$y = f(x), f(x) = \frac{x}{x^2 + 1}$$

## SOLUTION

$$f(x) = \frac{x}{x^2 + 1} \to Domain : x^2 + 1 \neq 0$$

$$x^2 + 1 \neq 0 \to x^2 \neq -1 \to x \in \mathbb{R}$$

Conclusion,

$$\boxed{f(x) = \frac{x}{x^2 + 1} \to Domain : x \in \mathbb{R}}$$

Consider the range.

The first method

$x^2 + 1 \geq 2x$ because $(x - 1)^2 \geq 0$,

$x^2 + 1 \geq -2x$ because $(x + 1)^2 \geq 0$.

Divide inequalities $x^2 + 1 \geq 2x$ and $x^2 + 1 \geq -2x$ by $x^2 + 1 > 0$.

Then

$$-\frac{1}{2} \leq \frac{x}{x^2 + 1} \leq \frac{1}{2}$$

The second method

$y = \frac{x}{x^2+1}$,

$yx^2 - x + y = 0$ is a quadratic equation in terms of $x$,

$D = 1 - 4y^2 \geq 0$,

Hence $y^2 \leq \frac{1}{4}$, that is, $-\frac{1}{2} \leq y \leq \frac{1}{2}$.

The third method.

1) We now find the asymptotic value of the function for $x \to \pm\infty$

$$\lim_{x\to+\infty} f(x) = \lim_{x\to+\infty} \frac{x}{x^2+1} = +0$$

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} \frac{x}{x^2+1} = -0$$

2) Let us find the extremum points

$$f(x) = \frac{x}{x^2+1} \to f'(x) = \frac{(x)' \cdot (x^2+1) - x \cdot (x^2+1)'}{(x^2+1)^2} = \frac{1 \cdot (x^2+1) - x \cdot (2x)}{(x^2+1)^2} =$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2} = \frac{1-x^2}{(1+x^2)^2} = \frac{(1-x)(1+x)}{(1+x^2)^2} \to \boxed{f'(x) = \frac{(1-x)(1+x)}{(1+x^2)^2}}$$

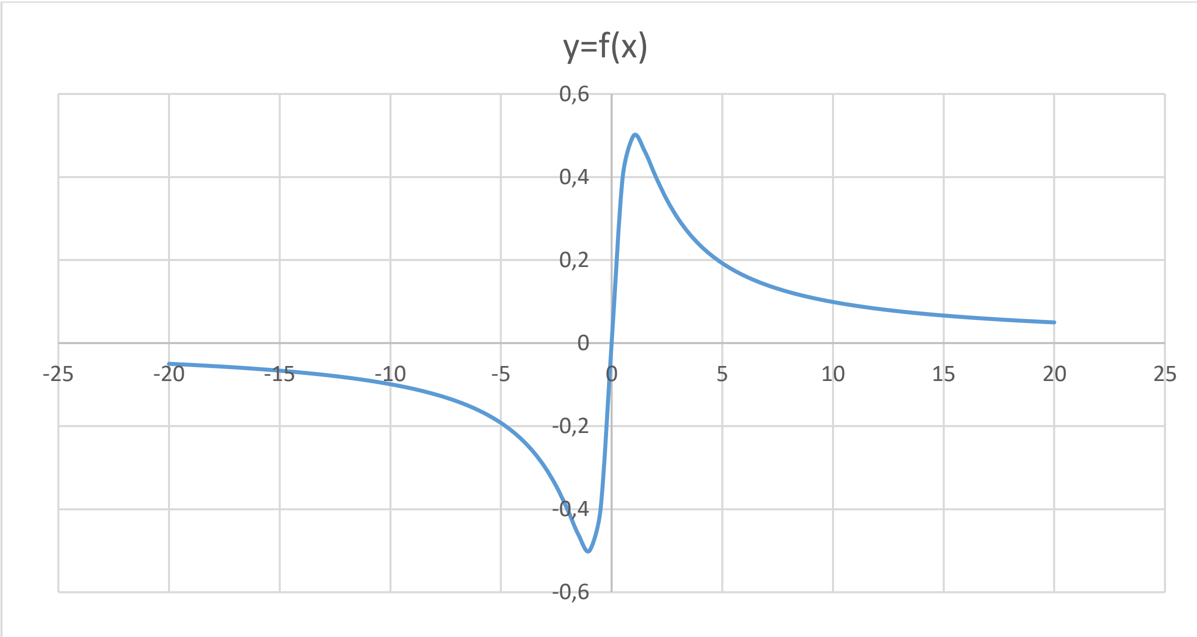
$$f'(x) = 0 \to \frac{(1-x)(1+x)}{(1+x^2)^2} = 0 \to \left[\begin{matrix} 1-x=0 \\ 1+x=0 \end{matrix}\right. \to \boxed{\left[\begin{matrix} x=1 \\ x=-1 \end{matrix}\right.}$$

| | $x \in (-\infty; -1)$ | $x = -1$ | $x \in (-1; 1)$ | $x = 1$ | $x \in (1; +\infty)$ |
|---|---|---|---|---|---|
| $f'(x)$ | $f'(x) < 0$ | $f'(x) = 0$ | $f'(x) > 0$ | $f'(x) = 0$ | $f'(x) < 0$ |
| $f(x)$ | *decreasing* | *local minimum* | *increasing* | *local maximum* | *decreasing* |

$$\min_{x\in\mathbb{R}} f(x) = f(-1) = \frac{-1}{(-1)^2+1} = -\frac{1}{2}$$

$$\max_{x\in\mathbb{R}} f(x) = f(1) = \frac{1}{1^2+1} = -\frac{1}{2}$$

3) Let's sketch this function using the received data




As we see from the sketch, the local extrema are global maxima and minima/

Then,

$$\boxed{y = f(x) = \frac{x}{x^2+1} \rightarrow Range: \ y \in \left[-\frac{1}{2}; \frac{1}{2}\right]}$$

**ANSWER:**

$$y = f(x), f(x) = \frac{x}{x^2+1} \rightarrow Domain: x \in \mathbb{R}$$

$y = f(x), f(x) = \frac{x}{x^2+1} \rightarrow Range: \ y \in \left[-\frac{1}{2}; \frac{1}{2}\right].$

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