Answer on Question #76068 – Math – Calculus

Question

Obtain the Fourier series for the following periodic function which has a period of 2π: f(x)=x^2 for−π≤x≤π

Solution

$f(x) = x^2$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n\cos(nx) + b_n\sin(nx))$

$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\, dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\, dx = \frac{1}{3\pi}x^3|_{x=-\pi}^{x=\pi} = \frac{2}{3}.$

$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\, dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\cos(nx)\, dx =$

$= \frac{1}{\pi}(\frac{2xcos(nx)}{n^2} + \frac{(n^2x^2 - 2)\sin(nx)}{n^3})|_{x=-\pi}^{x=\pi} = \frac{4}{n^2}\cos(\pi n) = \frac{4}{n^2}(-1)^n.$

$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\, dx = \frac{1}{\pi}\int_{-\pi}^{\pi} x^2\sin(nx)\, dx =$

$= \frac{1}{\pi}(\frac{2xsin(nx)}{n^2} - \frac{(n^2x^2-2)\cos(nx)}{n^3})|_{x=-\pi}^{x=\pi} = 0$

$f(x) = \frac{1}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nx)$.

Answer: $f(x) = \frac{1}{3} + 4\sum_{n=1}^{\infty}\frac{(-1)^n}{n^2}\cos(nx)$.