

Answer on Question #76062 – Differential Geometry | Topology

Question

The unit sphere S^2 defined by:

1) $\sigma(\theta, \varphi) = (\cos\theta\cos\varphi, \cos\theta\sin\varphi, \sin\theta)$

2) $\tilde{\sigma}(\theta, \varphi) = (-\cos\theta\cos\varphi, -\sin\theta, -\cos\theta\sin\varphi)$

Solution

Both functions define a set on the unit sphere, since $x^2 + y^2 + z^2 \equiv 1$ and $\tilde{x}^2 + \tilde{y}^2 + \tilde{z}^2 \equiv 1$. If the domain of the first function is R^2 , then its image is the whole unit sphere: variable $\frac{\pi}{2} - \varphi$ determines the angle between the Y -axis and the radius vector, and variable θ determines the angle between the projection of the radius vector onto the XZ -plane and the X -axis. If the domain of the second function is R^2 , then it also defines the sphere: $\tilde{x} = -x$, $\tilde{y} = -z$, $\tilde{z} = -y$.

Answer

Both these function define the unit sphere S^2 , if the domain of them is R^2 .