**Answer on Question #76040 – Math – Differential Equations**

**Question**

Find the general integral of the equation

$$(x-y)p+(y-x-z)q=z$$

and particular solution through the circle

$$z=1, x^2+y^2=1$$

**Solution**

Auxiliary equations:

$$\frac{dx}{x-y}=\frac{dy}{y-x-z}=\frac{dz}{z}$$

Then

$$\frac{dx+dy+dz}{x-y+y-x-z+z}=\frac{d(x+y+z)}{0}$$

$$d(x+y+z)=0$$

The 1st solution:

$$\varphi_1=x+y+z=c_1$$

$$\frac{dx-dy+dz}{x-y-y+x+z+z}=\frac{dz}{z}$$

$$\frac{d(x-y+z)}{x-y+z}=\frac{2dz}{z}$$

$$d\left(\ln\frac{x-y+z}{z^2}\right)=0$$

The 2nd solution:

$$\varphi_2=\frac{x-y+z}{z^2}=c_2$$

The general solution:

$$F(\varphi_1, \varphi_2) = F\left(x + y + z, \frac{x - y + z}{z^2}\right)$$

For $z = 1$:

$$\varphi_1 = x + y + z = x + y + 1 \ \Rightarrow x = \frac{\varphi_1 + \varphi_2}{2} - 1$$

$$\varphi_2 = \frac{x - y + z}{z^2} = x - y + 1 \ \Rightarrow y = \frac{\varphi_1 - \varphi_2}{2}$$

$$\begin{cases} x = \dfrac{\varphi_1 + \varphi_2}{2} - 1 \\ y = \dfrac{\varphi_1 - \varphi_2}{2} \end{cases}$$

For $x^2 + y^2 = 1$:

$$\left(\frac{\varphi_1 + \varphi_2}{2} - 1\right)^2 + \left(\frac{\varphi_1 - \varphi_2}{2}\right)^2 = 1$$

$$\left(\frac{\varphi_1 + \varphi_2}{2}\right)^2 - (\varphi_1 + \varphi_2) + 1 + \left(\frac{\varphi_1 - \varphi_2}{2}\right)^2 = 1$$

$$\frac{{\varphi_1}^2 + {\varphi_2}^2}{2} - (\varphi_1 + \varphi_2) = 0$$

$$\varphi_1(\varphi_1 - 2) + \varphi_2(\varphi_2 - 2) = 0$$

**Answer:**

$$(x + y + z)(x + y + z - 2) + \frac{x - y + z}{z^2}\left(\frac{x - y + z}{z^2} - 2\right) = 0$$