Answer on Question #76039 – Math – Differential Equations

Question

1.Find the differential equations of the space curve in which the two families of surfaces

u=x^2-y^2=c1 and v=y^2-z^2=c2 intersect.

2. Find value of n for which the equation (n-1)^2 u_xx-y^2n u_yy=ny^(2n-1) u_y is parabolic or hyperbolic.

Solution

1. $u = x^2 - y^2 = c_1,\ v = y^2 - z^2 = c_2.$

If (dx, dy, dz) are the projections of the tangent vector to the space curve in which the given surfaces intersect, then along any curve of the family, we have:

$$du = 0 \rightarrow 2xdx - 2ydy = 0 \rightarrow xdx = ydy.$$

$$dv = 0 \rightarrow 2ydy - 2zdz = 0 \rightarrow ydy = zdz.$$

Solving these two equations we get:

$\frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{xy}$ – differential equations of the space curve.

2. $(n-1)^2 u_{xx} - y^2 n u_{yy} = n y^{2n-1} u_y.$

$$D = b^2 - 4ac = 0 - 4(n-1)^2(-y^2) = 4(n-1)^2 y^2.$$

If $n \neq 1,\ D > 0$ – equation is hyperbolic.

If $n = 1,\ D = 0$ – equation is parabolic.