Answer on Question #75995 – Math – Discrete Mathematics

Question

X be a non-empty set and let R be an equivalence relation on X.
For each $x \in X$, define $[x] = \{y \in X \mid x\ R\ y\}$ to be the equivalence class of x.
Here x R y means $(x, y) \in R$.

Suppose that $A = [x]$ and $B = [y]$. Prove that if $A \cap B \neq \emptyset$, then $A = B$.

Solution

$A \cap B \neq \emptyset$. It means that $\exists\ z \in A \cap B$ i.e. $(z \in A) \wedge (z \in B) \sim (z\ R\ x) \wedge (z\ R\ y)$.
Because of the symmetry of the relation R $(x\ R\ z) \wedge (z\ R\ y)$. According to transitivity of the relation R we obtain that $x\ R\ y$.

1) We choose and fix $\forall\ h \in A$. Let's prove that $h \in B$.

$h \in A => (h\ R\ x) \wedge (x\ R\ y) => h\ R\ y\ \ i.e.\ h \in B$.

2) We choose and fix $\forall\ h \in B$. Similarly we can prove that $h \in A$.

$$h \in B => (h\ \ R\ y) \wedge (x\ R\ y) => (according\ to\ symmetry\ of\ R) =>$$

$$=> (h\ \ R\ y) \wedge (y\ R\ x) => (according\ to\ transitivity\ of\ R) => h\ R\ x,$$

i.e. $h \in A$ ■