Answer on Question #75987 – Math – Discrete Mathematics

Question

Prove by induction that the following statements are true for all integers n

a) 1^2x2+2^2x3+…..+n^2(n+1)= n(n+1)(n+2)(3n+1)/12

b) 4007^n-1 is divisible by 2003

Solution

a) $1^2 \times 2 + 2^2 \times 3 + \cdots + n^2(n+1) = n(n+1)(n+2)(3n+1)/12$

Let's prove this equality by induction on n:

1. n=1: $2 = 1^2 \times 2 = \frac{1\times2\times3\times4}{12} = \frac{24}{12} = 2.$
2. Assume that this equality holds for n=k:

   $1^2 \times 2 + 2^2 \times 3 + \cdots + k^2(k+1) = k(k+1)(k+2)(3k+1)/12$
3. Let's prove equality for n=k+1.

   $1^2 \times 2 + \cdots + (k+1)^2((k+1)+1) = 1^2 \times 2 + \cdots + (k^2+2k+1)(k+2) =$

   $= 1^2 \times 2 + \cdots + k^3 + 4k^2 + 5k + 2 = \frac{k(k+1)(k+2)(3k+1)}{12} + k^3 + 4k^2 +$

   $+5k + 2 = \frac{k(k+1)(k+2)(3k+1) + 12k^3 + 48k^2 + 60k + 24}{12} =$

   $= \frac{((k+1)+1)((k+1)+2)((k+1)+3)(3(k+1)+1)}{12} \Rightarrow$

   $\Rightarrow$ the equality is proved.

   The statement is true for all natural n by the principle of mathematical induction.

b) $4007^n - 1 \vdots 2003.$

Let's prove this equality by induction on n:

1. n=1. $4007^1 - 1 = 4006 \vdots 2003.$
2. Assume that this equality holds for n=k: $4007^k - 1 \ \vdots 2003$
3. Let's prove equality for n=k+1.

   $4007^{k+1} - 1 = 4007 \times 4007^k - 1 = 4006 \times 4007^k + 4007^k - 1.$

   1) $4006 \times 4007^k + 4007^k \vdots 2003;$

   2) $4007^k - 1 \vdots 2003 \Rightarrow 4006 \times 4007^k + 4007^k - 1 \ \vdots 2003.$

   $\Rightarrow$ the formula is proved.

   The statement is true for all natural n by the principle of mathematical induction.