

## Answer on Question #75946 – Math – Trigonometry

### Question

(1)  $\sin x \cos x = 0$

### Solution

$$\begin{aligned}\sin x \cos x &= 0 \\ \frac{1}{2} 2 \sin x \cos x &= 0 \\ \frac{1}{2} \sin 2x &= 0 \\ \sin 2x &= 0 \\ 2x &= \pi k, k \in \mathbb{Z} \\ x &= \frac{1}{2} \pi k, k \in \mathbb{Z}\end{aligned}$$

**Answer:**  $x = \frac{1}{2} \pi k, k \in \mathbb{Z}$

### Question

(2)  $2 \sin^2 x - \sin x - 1 = 0$

### Solution

$$\begin{aligned}2 \sin^2 x - \sin x - 1 &= 0 \\ 2 \sin^2 x - 2 \sin x + \sin x - 1 &= 0 \\ 2 \sin x (\sin x - 1) + (\sin x - 1) &= 0 \\ (2 \sin x + 1)(\sin x - 1) &= 0 \\ \text{a) } 2 \sin x + 1 = 0 \quad \vee \quad \text{b) } \sin x - 1 = 0\end{aligned}$$

a)  $2 \sin x + 1 = 0$

$$\begin{aligned}2 \sin x + 1 &= 0 \\ \sin x &= -\frac{1}{2} \\ x &= (-1)^{k+1} \frac{\pi}{6} + \pi k, k \in \mathbb{Z}\end{aligned}$$

b)  $\sin x - 1 = 0$

$$\begin{aligned}\sin x - 1 &= 0 \\ \sin x &= 1 \\ x &= \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}\end{aligned}$$

**Answer:**  $x = (-1)^{k+1} \frac{\pi}{6} + \pi k, k \in \mathbb{Z}$  and  $x = \frac{\pi}{2} + 2\pi k, k \in \mathbb{Z}$

### Question

(3)  $\cos x + \cos(2x) = 0$

### Solution

$$\begin{aligned}\cos x + \cos(2x) &= 0 \\ \cos x + 2 \cos^2 x - 1 &= 0 \\ (\cos x + 1)(2 \cos x - 1) &= 0\end{aligned}$$

$$a) \cos x + 1 = 0 \quad \vee \quad b) 2 \cos x - 1 = 0$$

$$a) \cos x + 1 = 0$$

$$\cos x = -1$$

$$x = \pi + 2\pi k, k \in \mathbb{Z}$$

$$b) 2 \cos x - 1 = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \pm \frac{\pi}{3} + 2\pi k, k \in \mathbb{Z}$$

$$\text{Answer: } x = \pi + 2\pi k, k \in \mathbb{Z} \text{ and } x = \pm \frac{\pi}{3} + 2\pi k, k \in \mathbb{Z}$$

### Question

$$(4) 2 \cos x + \sec x = 3$$

### Solution

$$2 \cos x + \sec x = 3$$

$$2 \cos x + \frac{1}{\cos x} = 3$$

$$2 \cos^2 x + 1 = 3 \cos x$$

$$2 \cos^2 x - 3 \cos x + 1 = 0$$

$$\text{Let } t = \cos x$$

$$2t^2 - 3t + 1 = 0$$

$$D = 9 - 8 = 1$$

$$t_1 = \frac{3-1}{4} = \frac{1}{2}$$

$$t_2 = \frac{3+1}{4} = 1$$

$$\text{So } a) \cos x = \frac{1}{2} \quad \text{or} \quad b) \cos x = 1$$

$$a) \cos x = \frac{1}{2}$$

$$x = \pm \frac{\pi}{3} + 2\pi k, k \in \mathbb{Z}$$

$$b) \cos x = 1$$

$$x = 2\pi k, k \in \mathbb{Z}$$

$$\text{Answer: } x = \pm \frac{\pi}{3} + 2\pi k, k \in \mathbb{Z}; x = 2\pi k, k \in \mathbb{Z}$$

### Question

$$(5) \sec x - 1 = \tan x$$

### Solution

$$\sec x - 1 = \tan x$$

$$\frac{1}{\cos x} - 1 = \tan x$$

$$1 - \cos x = \frac{\sin x}{\cos x} \cos x$$

$$\sin x + \cos x = 1$$

$$\frac{1}{\sqrt{2}} \sin x + \frac{1}{\sqrt{2}} \cos x = \frac{1}{\sqrt{2}}$$

$$\sin\left(x + \frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$$

$$x = -\frac{\pi}{4} + (-1)^k \frac{\pi}{4} + \pi k, k \in \mathbb{Z}$$

**Answer:**  $x = -\frac{\pi}{4} + (-1)^k \frac{\pi}{4} + \pi k, k \in \mathbb{Z}.$