Question #75766, Math / Differential Equations

Obtain the Fourier series expansion for the following periodic function which has a period of π: f(x)={(4/π) for 0<x<π/2

Answer.

$$f(x)=\begin{cases}\frac{4}{\pi}, & 0\le x<\pi/2\\ 0, & -\frac{\pi}{2}\le x<0\end{cases}$$

$$f(x)=\frac{a_0}{2}+\sum_{n=1}^{\infty}(a_n cosnx+b_n sinnx)$$

$$a_n=\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f(x)\,dx=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{4}{\pi}dx=\frac{4}{\pi^2}x\Big|_{x=0}^{x=\frac{\pi}{2}}=\frac{2}{\pi}.$$

$$a_n=\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f(x)cosnx\,dx=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{4}{\pi}cosnx\,dx=\frac{4}{\pi^2}\frac{1}{n}sinnx\Big|_{x=0}^{x=\frac{\pi}{2}}=\frac{4}{\pi^2}\frac{1}{n}sin\frac{\pi n}{2}$$

$$b_n=\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}f(x)sinnx\,dx=\frac{1}{\pi}\int_0^{\frac{\pi}{2}}\frac{4}{\pi}sinnx\,dx=-\frac{4}{\pi^2}\frac{1}{n}cosnx\Big|_{x=0}^{x=\frac{\pi}{2}}=$$

$$=-\frac{4}{\pi^2}\frac{1}{n}cos\frac{\pi n}{2}+\frac{4}{\pi^2 n}$$

$$f(x)=\frac{2}{\pi}+\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}\left[sin\frac{\pi n}{2}cosnx-\left(cos\frac{\pi n}{2}-1\right)sinnx\right]=$$

$$=\frac{2}{\pi}+\frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{1}{n}\left[sin\left(\frac{\pi n}{2}-nx\right)+sinnx\right]$$