Answer on Question #75682 – Math – Calculus

Question

Trace the curve x(y2+4)=8. stating all the points used while doing so

Solution

$$x(y^2+4)=8$$

i)

$$y^2=\frac{4(2-x)}{x} \Leftrightarrow y=\pm\sqrt{\frac{4(2-x)}{x}}$$

Note:

$$\begin{cases}\frac{4(2-x)}{x}\geq 0 \\ x>0\end{cases} \Leftrightarrow \begin{cases}4(2-x)\geq 0 \\ x>0\end{cases} \Leftrightarrow \begin{cases}x\leq 2 \\ x>0\end{cases} \Rightarrow x\in(0;2]$$

Domain of the function: $D(f)=(0;2]$

ii)

Take:

$$y=\sqrt{\frac{4(2-x)}{x}}$$

Range of the function: $E(y)=[0;\ +\infty)$ $\Rightarrow$ Range of the function $\left(y=\pm\sqrt{\frac{4(2-x)}{x}}\right)$: $(-\infty;\ \infty)$

Take

$$y=-\sqrt{\frac{4(2-x)}{x}}$$

Range of the function: $E(f)=(-\infty;0]$

iii)

Take:

$$y=\sqrt{\frac{4(2-x)}{x}}$$

If $f(-x)=f(x), then\ the\ function\ is\ even$

If $f(-x)=-f(x), then\ the\ function\ is\ odd$

$$F(-x)=\sqrt{\frac{4(2+x)}{-x}}=-\sqrt{\frac{4(2+x)}{x}} \Longrightarrow \text{the function is neither even nor odd}$$

Take

$$y=-\sqrt{\frac{4(2-x)}{x}}$$

$$F(-x) = -\sqrt{\frac{4(2+x)}{-x}} = +\sqrt{\frac{4(2+x)}{x}} \Longrightarrow \text{the function is neither even nor odd}$$

$$\Downarrow$$

$$y = \pm\sqrt{\frac{4(2-x)}{x}} - \text{the function is neither even nor odd}$$

iv)

If the function is periodic, then $f(x+T) = f(x)$

Obviously, there is a number $x \in D(f)$ such that the equality $f(x+T) = f(x)$ for our function is not satisfied.

$$\Downarrow$$

The function $y = \pm\sqrt{\frac{4(2-x)}{x}}$ is non-recurrent.

v) a) The point of intersection with the axis ox:

$$y = 0 \Rightarrow \pm\sqrt{\frac{4(2-x)}{x}} = 0$$

$$\frac{4(2-x)}{x} = 0 \Leftrightarrow \begin{cases} 2-x=0 \\ x \neq 0 \end{cases} \Leftrightarrow \{x = 2$$

The point of intersection with the axis ox is $A(2;0)$

b) The point of intersection with the axis oy:

$$x = 0 \Longrightarrow \pm\sqrt{\frac{4(2-x)}{x}} = \pm\sqrt{\frac{4(2-0)}{0}}$$

There are no solutions, since the denominator is zero.

The point of intersection with the axis oy does not exist.

c) The largest and smallest value of the function:

$$y' = \left(\sqrt{\frac{4(2-x)}{x}}\right)' = 2\left(\sqrt{\frac{(2-x)}{x}}\right)' = 2\left(\frac{1}{2\sqrt{\frac{2-x}{x}}}\right) * \left(\frac{2-x}{x}\right)' = 2\left(\frac{1}{2\sqrt{\frac{2-x}{x}}}\right) * \frac{2}{x^2} =$$

$$= \frac{-2}{x^2\sqrt{\frac{2-x}{x}}} = \frac{-2}{x^2\sqrt{\frac{2}{x}-1}}$$

Likewise

$$y' = -\left(\sqrt{\frac{4(2-x)}{x}}\right)' = \frac{2}{x^2\sqrt{\frac{2}{x}-1}}$$

$$\frac{2}{x^2\sqrt{\frac{2}{x}-1}} = 0 \qquad and \qquad \frac{-2}{x^2\sqrt{\frac{2}{x}-1}} = 0$$

$x \in \emptyset$ $\qquad$ $x \in \emptyset$

The largest and smallest value of the function does not exist

vi) Intervals of increasing and decreasing function:

Take:

$$y = \sqrt{\frac{4(2-x)}{x}}$$

$$y' = \left(\sqrt{\frac{4(2-x)}{x}}\right)' = -\frac{2}{x^2\sqrt{\frac{2}{x}-1}}$$

$$-\frac{2}{x^2\sqrt{\frac{2}{x}-1}} < 0 \ \Rightarrow \ x^2\sqrt{\frac{2}{x}-1} > 0 \ \Rightarrow x \in (0;2)$$

At $x = 2$, the function is defined and continuous, so it should be added to both the ascending and descending intervals.

The interval of decreasing function $x \in (0;\ 2]$

$$-\frac{2}{x^2\sqrt{\frac{2}{x}-1}} > 0 \ \Rightarrow \ \frac{2}{x^2\sqrt{\frac{2}{x}-1}} < 0 \ \Rightarrow x \in \emptyset$$

Ascending intervals do not exist.

Take

$$y = -\sqrt{\frac{4(2-x)}{x}}$$

$$y' = -\left(\sqrt{\frac{4(2-x)}{x}}\right)' = \frac{2}{x^2\sqrt{\frac{2}{x}-1}}$$

$$\frac{2}{x^2\sqrt{\frac{2}{x}-1}} > 0 \ \Rightarrow x^2\sqrt{\frac{2}{x}-1} > 0 \ \Rightarrow \ x \in (0;2)$$

At $x = 2$, the function is defined and continuous, so it should be added to both the ascending and descending intervals.

The interval of increasing functions of $x \in (0;\ 2]$

$$\frac{2}{x^2\sqrt{\frac{2}{x}-1}} < 0 \Rightarrow x^2\sqrt{\frac{2}{x}-1} < 0 \Rightarrow \quad x \in \emptyset$$

Descending intervals do not exist.

$$\Downarrow$$

On the interval $x \in (0;\ 2]$ in the first quarter the function $y = \pm\sqrt{\frac{4(2-x)}{x}}$ decreases.

On the interval $x \in (0;\ 2]$ in the fourth quarter the function increases.

Note: the function is symmetric about the ox axis

vii)

$$f(x) == \pm\sqrt{\frac{4(2-x)}{x}}$$

Denominator goes to zero when $x = 0$

Take: $$y = \sqrt{\frac{4(2-x)}{x}}$$

$$\lim_{x\to 0^+}\sqrt{\frac{4(2-x)}{x}} = \lim_{x\to 0^+}\sqrt{\frac{4(2-0)}{+0}} = \lim_{x\to 0^+}\sqrt{\frac{8}{+0}} = +\infty$$

$$\lim_{x\to 0^-}\sqrt{\frac{4(2-x)}{x}} = \lim_{x\to 0^-}\sqrt{\frac{4(2+0)}{-0}} = \lim_{x\to 0^-}\sqrt{\frac{8}{-0}} = i\infty$$

$$\Downarrow$$

One-sided limits are infinite, so the line x=0 is the vertical asymptote of the graph of the function at $x \longrightarrow 0$.

Take $$y = -\sqrt{\frac{4(2-x)}{x}}$$

Likewise:

$$\lim_{x \to 0^+} -\sqrt{\frac{4(2-x)}{x}} = -\infty$$

$$\lim_{x \to 0^-} -\sqrt{\frac{4(2-x)}{x}} = (-i)\infty$$

$$\Downarrow$$

One-sided limits are infinite, so the line x=0 is the vertical asymptote of the graph of the function at $x \longrightarrow 0$.

viii)

The inflection points of a function:

$$y'' = \pm\left(\sqrt{\frac{4(2-x)}{x}}\right)'' = \pm\left(\frac{2}{x^2\sqrt{\frac{2}{x}-1}}\right)' = \pm\frac{6-4x}{\sqrt{\frac{2}{x}-1}(x-2)x^3}$$

Take:

$$y = -\sqrt{\frac{4(2-x)}{x}}$$

$$\frac{6-4x}{\sqrt{\frac{2}{x}-1}(x-2)x^3} = 0 \Rightarrow 6-4x = 0 \Rightarrow 4x = 6 \Rightarrow x = 1.5$$

changes the concavity at the point $x = 1.5$

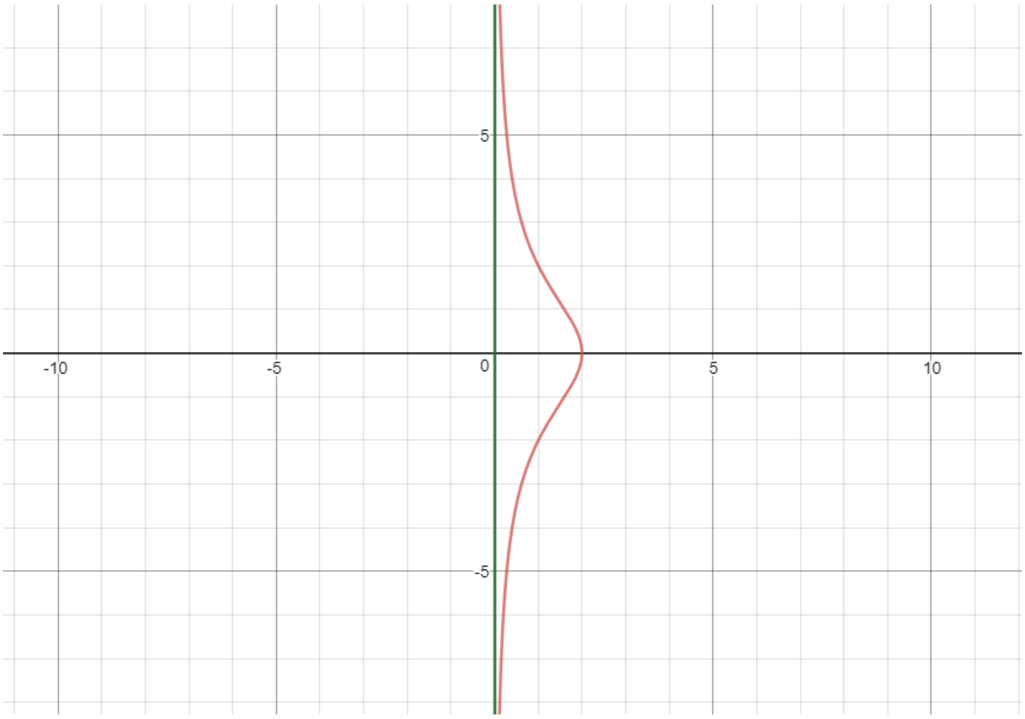
Take

$$y = \sqrt{\frac{4(2-x)}{x}}$$

$$-\frac{6-4x}{\sqrt{\frac{2}{x}-1}(x-2)x^3} = 0 \Rightarrow x = 1.5$$

changes the concavity at the point $x = 1.5$

xi)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