Answer on Question #75577 – Math – Trigonometry

Question

(1) Evaluate exactly $tan\left(Arccos\ \frac{2}{3}\right)$

(2) Evaluate exactly $cos\left(Arcsin\ 1 + Arccos\ \frac{1}{2}\right)$

(3) Determine $\arcsin\frac{1}{\sqrt{2}}$. Careful: that is arcsin and not Arcsin, so there are going to be infinitely many answers. You need to give all of them.

(4) Evaluate exactly $sin\left(\frac{1}{2}Arcsin\frac{4}{5}\right)$

(5) Use a calculator to determine $Arctan\ 10$ in radians to two decimal places.

Solution

1. Formula: $tan(Arccos\ x) = \frac{\sqrt{1-x^2}}{x}, |x| \le 1, x \ne 0.$

$$tan\left(Arccos\ \frac{2}{3}\right) = \frac{\sqrt{1-\left(\frac{2}{3}\right)^2}}{\frac{2}{3}} = \frac{\sqrt{1-\frac{4}{9}}}{\frac{2}{3}} = \frac{\sqrt{\frac{5}{9}}}{\frac{2}{3}} = \frac{\sqrt{5}}{3} * \frac{3}{2} = \frac{\sqrt{5}}{2}.$$

2. Formula: $cos(\alpha + \beta) = cos\alpha\ cos\beta - sin\alpha\ sin\beta.$

$$cos\left(Arcsin\ 1 + Arccos\ \frac{1}{2}\right) = cos(Arcsin\ 1)\ cos\left(Arccos\ \frac{1}{2}\right) - sin(Arcsin\ 1)\ sin\left(Arccos\ \frac{1}{2}\right) =$$
$$= cos\frac{\pi}{2} * \frac{1}{2} - 1 *\ sin\frac{\pi}{3} = 0 * \frac{1}{2} - 1 *\ \frac{\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$$

3. Formula: $\arcsin a\ = (-1)^n \operatorname{Arcsin} a \pm \pi n,\ \ n \in Z.$

$$\arcsin\frac{1}{\sqrt{2}} = (-1)^n\frac{\pi}{4} \pm \pi n, n \in Z.$$

4. Formula: $sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$; $\cos(Arcsin\ x) = \sqrt{1-x^2}$

$$sin\left(\frac{1}{2}Arcsin\frac{4}{5}\right) = \pm\sqrt{\frac{1-\cos\left(Arcsin\ \frac{4}{5}\right)}{2}} = \pm\sqrt{\frac{1-\sqrt{1-\left(\frac{4}{5}\right)^2}}{2}} = \pm\sqrt{\frac{1-\sqrt{\frac{9}{25}}}{2}} = \pm\sqrt{\frac{1-\frac{3}{5}}{2}} =$$
$$= \pm\sqrt{\frac{1}{5}} = \pm\frac{\sqrt{5}}{5}$$

5. $Arctan\ 10 = 1.471127 \approx 1.47$ radian.

Answer: 1. $\frac{\sqrt{5}}{2}$; 2. $-\frac{\sqrt{3}}{2}$; 3. $(-1)^n \frac{\pi}{4} \pm \pi n, n \in Z$; 4. $\pm \frac{\sqrt{5}}{5}$; 5. 1.47 radian.