Answer on Question #75562 – Math – Differential Geometry | Topology

Question

Show that the circular cylinder S={(x,y,z)∈R^3 |x^2+y^2=1} can be covered by a single surface patch and so a surface.

Solution

We can take U an annulus instead of a disc, where $U = \{(u, v): 0 < u^2+v^2 < \pi\}$. Any point in the annulus U is uniquely of the form $(t\cos\vartheta, t\sin\vartheta)$ for some real $t \in (0,\sqrt{\pi})$, $\vartheta \in [0,2\pi)$. Map this point to the point of the cylinder $(x, y, z) = (\cos\vartheta, \sin\vartheta, \cot(t^2))$.This is clearly a subset of the cylinder as it satisfies $x^2+y^2=1$. Also, because ϑ ranges in $[0,2\pi)$, for any fixed z the entire slice of the cylinder at that z level gets covered. Finally, because the cotangent of t^2 for $t \in (0,\sqrt{\pi})$ takes on *every* real value, every level z indeed gets a hit, showing the result of mapping the annulus as above covers the whole cylinder.