

## Answer on Question #75522 – Math – Quantitative Methods

### Question

Consider the following data

x	f(x)
1.0	0.7651977
1.3	0.6200860
1.6	0.4554022
1.9	0.2818186
2.2	0.1103623

Use Stirling's formula to approximate  $f(1.5)$  with  $x_0 = 1.6$

### Solution

First divided differences

$$\frac{0.6200860 - 0.7651977}{1.3 - 1.0} = -0.4837057$$

$$\frac{0.4554022 - 0.6200860}{1.6 - 1.3} = -0.5489460$$

$$\frac{0.2818186 - 0.4554022}{1.9 - 1.6} = -0.5786120$$

$$\frac{0.1103623 - 0.2818186}{2.2 - 1.9} = -0.571521$$

Second divided differences

$$\frac{-0.5489460 - (-0.4837057)}{1.6 - 1.0} = -0.1087338$$

$$\frac{-0.5786120 - (-0.5489460)}{1.9 - 1.3} = -0.0494433$$

$$\frac{-0.0494433 - (-0.5786120)}{2.2 - 1.6} = 0.0118183$$

Third divided differences

$$\frac{-0.0494433 - (-0.1087338)}{1.9 - 1.0} = 0.0658783$$

$$\frac{0.0118183 - (-0.0494433)}{2.2 - 1.3} = 0.0680684$$

Fourth divided differences

$$\frac{0.0680684 - (0.0658783)}{2.2 - 1.0} = 0.0018251$$

To apply Stirling's formula we use the underlined entries in the difference

x	F(x)	First divided differences	Second divided differences	Third divided differences	Fourth divided differences
1.0	0.7651977				
		-0.4837057			
1.3	0.6200860		-0.1087338		
		<u>-0.5489460</u>		<u>0.0658783</u>	
1.6	<u>0.4554022</u>		<u>-0.0494433</u>		<u>0.0018251</u>
		<u>-0.5786120</u>		<u>0.0680684</u>	
1.9	0.2818186		0.0118183		
		-0.5715210			
2.2	0.1103623				

The Stirling's formula, with  $h = 0.3$ ,  $x_0 = 1.6$  and  $s = -\frac{1}{3}$ , becomes

$$\begin{aligned} f(1.5) &= 0.4554022 + \left(-\frac{1}{3}\right) \left(\frac{0.3}{2}\right) ((-0.5489460) + (-0.5786120)) + \left(-\frac{1}{3}\right)^2 (0.3)^2 (-0.0494433) \\ &\quad + \frac{1}{2} \left(-\frac{1}{3}\right) \left(\left(-\frac{1}{3}\right)^2 - 1\right) (0.3)^3 (0.0658783 + 0.0680684) \\ &\quad + \left(-\frac{1}{3}\right)^2 \left(\left(-\frac{1}{3}\right)^2 - 1\right) (0.3)^4 (0.0018251) = 0.5118200 \end{aligned}$$

**Answer:** 0.5118200.