Answer on Question #75453 - Subject – Abstract Algebra

Given: $G = < x >$ and $o(G) = 25$

To prove or disprove: $G = < x^{\alpha} >$ where α is a factor of 25.

Solution: Consider $G = < x >$ and $o(G) = 25$

$\Rightarrow$ G is a cyclic group and generated by x.

$\therefore \quad x^{25} = e \quad$ and $\quad o(x) = 25$

$\therefore \quad$ The order of an element in G can be 1, 5 or 25.

Let x^{α} generate the group G and α is a factor of 25, therefore

$o(x^{\alpha}) = 25 \quad$ and $\quad 25 = \alpha k$,where k <25 is an integer

Q $\quad x^{25} = e \quad \Rightarrow \quad x^{\alpha k} = e$

$\Rightarrow \quad (x^{\alpha})^k = e$

$\Rightarrow \quad k = 25$

But $\quad k \neq 25$. Therefore α can't be a factor of 25.

Hence, given statement is false.

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