Answer on Question #75371– Math – Analytic Geometry

Question

Obtain the equation of the sphere having center on the line x/3= y/2= z/-5 and passing through the points (0,-2,-4) and (2,-1,-1).

Solution

Using the formula of the equation of the sphere

$$(x-x_0)^2+(y-y_0)^2+(z-z_0)^2=R^2,$$

where (x_0, y_0, z_0) —the center of the sphere, R – radius of the sphere.

As the sphere passing through the points (0,-2,-4) and (2,-1,-1) then the distance from the center to each of these points – radius of the sphere. Using the formula of the distance between two points we will find the radius:

$$R=\sqrt{(x_0-0)^2+(y_0+2)^2+(z_0+4)^2}$$

or (1)

$$R=\sqrt{(x_0-2)^2+(y_0+1)^2+(z_0+1)^2}$$

let's equate the right parts:

$$\sqrt{(x_0-0)^2+(y_0+2)^2+(z_0+4)^2}=\sqrt{(x_0-2)^2+(y_0+1)^2+(z_0+1)^2}$$

Then

$$(x_0-0)^2+(y_0+2)^2+(z_0+4)^2=(x_0-2)^2+(y_0+1)^2+(z_0+1)^2$$

$${x_0}^2+{y_0}^2+4y_0+4+{z_0}^2+8z_0+16={x_0}^2-4x_0+4+{y_0}^2+2y_0+1+{z_0}^2+2z_0+1=$$

$$4x_0+2y_0+6z_0+14=0. \quad (2)$$

As the sphere having center on the line x/3= y/2= z/-5 then the point (x_0, y_0, z_0) belongs to the line. Thus:

$$\frac{x_0}{3}=\frac{y_0}{2}, \frac{x_0}{3}=-\frac{z_0}{5}.$$

or

$$y_0=\frac{2x_0}{3}, z_0=-\frac{5x_0}{3}. \quad (3)$$

Let's substitute in (2):

$$4x_0+2\cdot\frac{2x_0}{3}+6\left(-\frac{5x_0}{3}\right)+14=0,$$

$$\frac{12x_0+4x_0-30x_0}{3}=-14,$$

$$\frac{-14x_0}{3}=-14, x_0=3.$$

Substitute in (3):

$$y_0=\frac{2\cdot 3}{3}=2, z_0=-\frac{5\cdot 3}{3}=-5,$$

then the point (3,2,-5) the center of the sphere

Substitute in (1):

$$R = \sqrt{(3-0)^2 + (2+2)^2 + (-5+4)^2} = \sqrt{26}$$

Thus the equation of the sphere is

$$(x-3)^2 + (y-2)^2 + (z+5)^2 = 26.$$

Answer: $(x-3)^2 + (y-2)^2 + (z+5)^2 = 26.$