Answer on Question #75362 – Math – Quantitative Methods

Question

Write down Runge Kutta Fehlberg Method(Error Control)

Solution

Consider the problem:

$$y' = f(x, y), \qquad y(x_0) = y_0$$

The Runge-Kutta-Fehlberg method is single-step method. This method has a procedure to determine whether the correct step size h is used. Each step requires the use of the following six values:

$$k_1 = h \cdot f(x_k, y_k)$$

$$k_2 = h \cdot f\left(x_k + \frac{1}{4}h, y_k + \frac{1}{4}k_1\right)$$

$$k_3 = h \cdot f\left(x_k + \frac{3}{8}h, y_k + \frac{3}{32}k_1 + \frac{9}{32}k_2\right)$$

$$k_4 = h \cdot f\left(x_k + \frac{12}{13}h, y_k + \frac{1932}{2197}k_1 - \frac{7200}{2197}k_2 + \frac{7296}{2197}k_3\right)$$

$$k_5 = h \cdot f\left(x_k + h, y_k + \frac{439}{216}k_1 - 8k_2 + \frac{3680}{513}k_3 - \frac{845}{4104}k_4\right)$$

$$k_6 = h \cdot f\left(x_k + \frac{1}{2}h, y_k - \frac{8}{27}k_1 + 2k_2 - \frac{3544}{2565}k_3 + \frac{1859}{4104}k_4 - \frac{11}{40}k_5\right)$$

Then we calculate the approximation to the solution with the help of the method of the fourth order:

$$w_{k+1} = y_k + \frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4101}k_4 - \frac{1}{5}k_5, \qquad Error = O(h^4)$$

And the approximation to the solution with the help of the method of the 5th order:

$$y_{k+1} = y_k + \frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 - \frac{9}{50}k_5 + \frac{2}{55}k_6, \qquad Error = O(h^5)$$

At each step, two different approximations for the solution are made and compared.

$$R = \frac{1}{h}|w_{k+1} - y_{k+1}|, \qquad \delta = \left(\frac{\varepsilon}{2R}\right)^{\frac{1}{4}}$$

If the two answers are in close agreement ($R \le \varepsilon$), the approximation is accepted. If the two answers do not agree to a specified accuracy (ε), the step size is reduced. If the answers agree to more significant digits than required, the step size is increased. The optimal step size is ($\delta \cdot h$)