Given tan(A) = (3/4), 0 < A < (π/2) and cos(B) = (5/13), (3π/2) < B < 2π) determine sin(A - B)

Solution

Solution

Let find $\cos A$. We will use formula:

$$1 + tan^2A = \frac{1}{cos^2A}$$

$$cos^2A = \frac{1}{1 + tan^2A}$$

$$cos^2A = \frac{1}{1 + (\frac{3}{4})^2} = \frac{1}{1 + \frac{9}{16}} = \frac{1}{\frac{16 + 9}{16}} = \frac{16}{25}$$

$$cosA = \pm\frac{4}{5}$$

If 0<A<$\frac{\pi}{2}$ then $cosA = \frac{4}{5}$

Let find sin$A$

We will use the formula:

$$sin^2A + cos^2A = 1$$

$$sin^2A = 1 - cos^2A = 1 - (\frac{4}{5})^2 = \frac{25 - 16}{25} = \frac{9}{25}$$

$$sinA = \pm\frac{3}{5}$$

If 0<A<$\frac{\pi}{2}$ then $sinA = \frac{3}{5}$

Let find sin$B$

We will use the formula:

$$sin^2B + cos^2B = 1$$

$$sin^2B = 1 - cos^2B = 1 - (\frac{5}{13})^2 = \frac{169 - 25}{169} = \frac{144}{169}$$

$$sinB = \pm\frac{12}{13}$$

If $\frac{3\pi}{2}$<B<2π then $sinB = -\frac{12}{13}$

Let find sin(A-B). We will use the formula:

Sin(A-B)=sinAcosB-cosAsinB

Sin(A-B)=$\frac{3}{5} \cdot \frac{5}{13} - \frac{4}{5} \cdot \left(-\frac{12}{13}\right) = \frac{3}{13} + \frac{48}{65} = \frac{15+48}{65} = \frac{63}{65}$

Answer

Sin(A-B)=$\frac{63}{65}$