Answer to Question #75039, Math / Statistics and Probability

Task

For a Poisson distributed random variable X, P(X = 4) = P(X = 5). What is the mean and variance of the distribution.

Solution

Probability mass function of Poisson distribution is following:

$$P(x = k) = \frac{\lambda^k e^{-\lambda}}{k!}$$

Where, k is the number of occurrences and λ is the expected number of occurrences.

$$P(x = 4) = \frac{\lambda^4 e^{-\lambda}}{4!} = \frac{\lambda^4 e^{-\lambda}}{24}$$

$$P(x = 5) = \frac{\lambda^5 e^{-\lambda}}{5!} = \frac{\lambda^5 e^{-\lambda}}{120}$$

$$P(x = 4) = P(x = 5) =>$$

$$\frac{\lambda^4 e^{-\lambda}}{24} = \frac{\lambda^5 e^{-\lambda}}{120} =>$$

$$\frac{1}{24} = \frac{\lambda}{120} =>$$

$$\lambda = \frac{24}{120} = \frac{1}{5}$$

Mean and Variance of Poisson distribution equals to the expected number of occurences λ.

E(X) = $\lambda = \frac{1}{5}$

D(X) = $\lambda = \frac{1}{5}$

Answer

E(X) = $\mathrm{D(X)} = \frac{1}{5}$