Question #7503 Solve $2dy/dx + 6y/x = 2\tan(x^4)$.
Solution. It is first order linear inhomogeneous equation. We are to use method, which is called variation of constants. First, look at the corresponding homogeneous equation: $2y' + 6y/x = 0$ or $y' = -3y/x$, which is equivalent to $dy/y = -3dx/x$, hence $\log|y| = -3\log|x| + C$, so $y = C \cdot x^{-3}$, where $C \in \mathbb{R}$. We must find the general solution of our equation in the form $y(x) = C(x) \cdot x^{-3}$, where $C(x)$ is a differentiable function. Substituting the last to our equation, one can get $2c'(x)x^{-3} - 6c(x)x^{-4} + 6c(x)x^{-4} = 2\tan x^4$ or $c'(x) = x^3 \tan x^4$. Integrating the last $c(x) = \int x^3 \tan x^4\, dx = |z = x^4,\ dz = 4x^3 dx| = \frac{1}{4}\int \tan z\, dz = -1/4 \log|\cos z| + C_1 = -1/4 \log|\cos x^4| + C_1$, where C_1 is arbitrary real constant. Hence, the general solution of our equation is $y(x) = x^{-3} \cdot (-1/4 \log|\cos x^4| + C_1)$, where $C_1 \in \mathbb{R}$.
Answer. $y(x) = x^{-3} \cdot (-1/4 \log|\cos x^4| + C_1)$, where $C_1 \in \mathbb{R}$.