Answer on Question #75021-Math-Other

A blow-moulded polymer container can be considered as a cylinder with flat ends. Its capacity is 2 litre and it has thin walls of uniform thickness.

(a) Produce expressions for its volume and surface area.

(b)Using differential calculus, find the dimensions of the cylinder which will result in the minimum amount of polymer being used for its manufacture.

Solution

$$1\ L = 1\ dm^3.$$

(a) The volume is

$$V = \pi R^2 H = 2\ L.$$

The surface area is

$$A = 2(\pi R^2) + 2\pi RH = 2\pi R(R + H).$$

(b)

$$H = \frac{V}{\pi R^2} = \frac{2}{\pi R^2}$$

$$A = 2\pi R\left(R + \frac{2}{\pi R^2}\right) = 2\pi R^2 + \frac{4}{R}.$$

$$\frac{dA}{dt} = 4\pi R - \frac{4}{R^2} = 0$$

$$4\pi R^3 = 4$$

The radius of the cylinder is

$$R = \frac{1}{\sqrt[3]{\pi}}\ dm \approx 0.683\ dm = 6.83\ cm.$$

The height of the cylinder is

$$H = \frac{2}{\pi R^2} = \frac{2}{\pi\left(\frac{1}{\sqrt[3]{\pi}}\right)^2} = \frac{2}{\sqrt[3]{\pi}}dm \approx 1.366\ dm = 13.66\ cm.$$