Question #7501 Solve $x(x+y)dy/dx = y(x-y)$.
Solution. This equation is homogeneous(it does not change when we make the substitution $x \mapsto \lambda x\,, y \mapsto \lambda y$). Hence, it is reasonable to make a substitution $y = zx$, $y' = z'x + z$, thus our equation is equivalent to

$$\frac{x}{x+xz}(z'x+z) = \frac{zx}{x-zx}, \text{ or } \frac{1}{1+z}(z'x+z) = \frac{z}{1-z}.$$

We get $z' \cdot x = \dfrac{z(1+z)}{1-z} - z = \dfrac{z+z^2-z+z^2}{1-z}$, finally we obtain $1/2dz\dfrac{1-z}{z^2} = dx/x$. Or $1/2(-1/z - \log|z|) = \log|x| + C$, returning back to initial variables $1/2(-\dfrac{x}{y} + \log|x| - \log|y|) = \log|x| + C$.
Answer. The general solution can be obtained from the relation $-\dfrac{x}{y} - \log|y| = \log|x| + C$, $C \in \mathbb{R}$ is arbitrary real constant.