Question #7500 Solve $(\cos x + \log y) + dy/dx = -(x/y + e^y)dy/dx$.

Solution. Denote $M(x, y) := \cos + \log y$, $N(x, y) := x/y + e^y + 1$. Our equation is equivalent to $(\cos x + \log y)dx + (x/y + e^y + 1)dy = 0$. Due to $\frac{\partial M(x,y)}{\partial y} = \frac{\partial N(x,y)}{\partial x} = \frac{1}{y}$ our DE is exact. Denote by $U(x, y)$ such function that $U(x, y) = 0$ defines the general solution. U satisfies $\frac{\partial U}{\partial x} = \cos x + \log y$, thus $U(x, y) = \sin x + (\log y)x + \varphi(y)$. One can get $\frac{\partial U}{\partial y} = x/y + \varphi'(y) = N(x, y) = x/y + e^y + 1$, hence $\varphi'(y) = e^y + 1$, so $\varphi(y) = e^y + y + C$. $U(x, y) = \sin x + \log y \cdot x + e^y + y + C$, and the general solution is obtained from $U(x, y) = 0$

Answer. The general solution $\sin x + \log y \cdot x + e^y + y + C = 0$