ANSWER on Question #74996 – Math – Calculus

$2\ kg$ of water is heated from $0°C$ to $100°C$ and converted into steam at the same

temperature. Calculate the increase in entropy, given that specific heat of water is

$$c = 4.18 \times 10^3 \frac{J}{kg \cdot K} \to c = 4180 \frac{J}{kg \cdot K}$$

and Latent heat of vaporization is

$$L_{vaporization} = 2.27 \times 10^7 \frac{J}{kg}$$

SOLUTION

Entropy is a state function defined by (per unit mass)

$$ds = \frac{dq_{rev}}{T}$$

The second law defines entropy as a state function and permits the following statements:
a) For a reversible process the entropy of the universe remains constant.

b) For an irreversible process the entropy of the universe will increase.

(More information: https://en.wikipedia.org/wiki/Entropy)

In our case,

Entropy increases in other processes: 1) heating of water; 2) water transfer to steam.

$$\Delta S = \Delta S_1 + \Delta S_2$$

Step 1: Compute the increase in entropy resulting from increasing the water temperature from $0°C$ to $100°C$:

Let us first translate the temperature into the absolute Kelvin scale

$$t_1 = 0°C \to T_1 = (0 + 273)K = 273K$$

$$t_2 = 100°C \to T_2 = (100 + 273)K = 373K$$

Then,

$$ds = \frac{cmdT}{T} \to \int_{s_1}^{s_2} ds = \int_{T_1}^{T_2} \frac{cmdT}{T} \to \underbrace{s_2 - s_1}_{\Delta S_1} = cm \cdot \ln|T| |_{T_1}^{T_2} = cm \cdot (\ln|T_2| - \ln|T_1|) \to$$

$$\Delta S_1 = cm \cdot \ln\left(\frac{T_2}{T_1}\right) = 4180 \frac{J}{kg \cdot K} \cdot 2kg \cdot \ln\left(\frac{373}{273}\right) = \left(8360 \cdot \ln\left(\frac{373}{273}\right)\right) \frac{J}{K} \to$$

$$\boxed{\Delta S_1 = \left(8360 \cdot \ln\left(\frac{373}{273}\right)\right) \frac{J}{K} \approx 2609.211 \frac{J}{K}}$$

Step 2: Compute the change in entropy from conversion of $2kg$ of water to steam, which involves a latent heat term. This is

$$\Delta S_2 = \frac{mL_{vaporization}}{T} = \frac{2kg \cdot 2.27 \times 10^7 \frac{J}{kg}}{373K} = \frac{4.54 \times 10^7}{373} \frac{J}{K} \approx 121715.818 \frac{J}{K}$$

$$\boxed{\Delta S_2 = \frac{4.54 \times 10^7}{373} \frac{J}{K} \approx 121715.818 \frac{J}{K}}$$

Conclusion,

$$\Delta S = \Delta S_1 + \Delta S_2 \approx 2609.211 \frac{J}{K} + 121715.818 \frac{J}{K} = 124325.029 \frac{J}{K}$$

$$\boxed{\Delta S \approx 124325.029 \frac{J}{K}}$$

ANSWER

$$\Delta S \approx 124325.029\frac{J}{K}$$