Answer to Question #74983, Math / Statistics and Probability

X is a random variable taking values 0 and 1 with respective probabilities q and p. A sample $X_1, X_2, \dots, X$ of size n is taken from the distribution. If $r = 1, x$,, show that the bias of the estimator tends to zero as n tends to infinity.

Solution.

We have Bernoulli distribution with the probability mass function:

$$f(x_i, p) = p^{x_i}(1-p)^{1-x_i} \ for\ x_i \in \{0,1\}$$

Then the bias of the estimator:

$$B(\hat{p}) = E(\hat{p}) - p$$

With estimator:

$$\hat{p} = \frac{\sum_{i=1}^{n} x_i}{n}$$

Variance of estimator:

$$var(\hat{p}) = var\left(\frac{\sum_{i=1}^{n} x_i}{n}\right) = \frac{1}{n^2}\sum_{i=1}^{n} var(x_i) = \frac{1}{n^2}\sum_{i=1}^{n}(pq) = \frac{npq}{n^2} = \frac{pq}{n}$$

$$\lim_{n\to\infty} var(\hat{p}) = \lim_{n\to\infty}\frac{pq}{n} = 0$$