

ANSWER on Question #74956 – Math – Trigonometry

For what values of x , $0 \leq x < 2\pi$ is it true that

$$\csc x = \sqrt{(\cot^2 x + 1)}?$$

a. $0 < x < \pi$;

b. $\pi < x < 2\pi$;

c. $\frac{\pi}{2} \leq x \leq \frac{3\pi}{2}$;

d. $0 \leq x < \frac{\pi}{2}$, $\pi < x < \frac{3\pi}{2}$;

e. None of these.

SOLUTION

Recall some definitions of trigonometry

$$\cot x = \frac{\cos x}{\sin x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\sin x \geq 0 \quad \forall x \in [2\pi n; \pi + 2\pi n], \quad n \in \mathbb{Z}$$

(More information: https://en.wikipedia.org/wiki/List_of_trigonometric_identities)

$$\sin^2 x + \cos^2 x = 1, \quad \forall x \in \mathbb{R}$$

(More information: https://en.wikipedia.org/wiki/Pythagorean_trigonometric_identity)

Just remember the definition of the number modulus

$$|a| = \begin{cases} a, & a \geq 0 \\ -a, & a < 0 \end{cases}$$

(More information: https://en.wikipedia.org/wiki/Absolute_value)

Just remember the properties of the square root

$$\sqrt{x^2} = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

(More information: https://en.wikipedia.org/wiki/Square_root)

In our case,

$$\begin{aligned} \sqrt{(\cot^2 x + 1)} &= \sqrt{\left(\left(\frac{\cos x}{\sin x}\right)^2 + 1\right)} = \sqrt{\frac{\cos^2 x}{\sin^2 x} + 1} = \sqrt{\frac{\cos^2 x}{\sin^2 x} + \frac{\sin^2 x}{\sin^2 x}} = \sqrt{\frac{\overbrace{\cos^2 x + \sin^2 x}^{=1}}{\sin^2 x}} = \\ &= \sqrt{\frac{1}{\sin^2 x}} = \left|\frac{1}{\sin x}\right| = \frac{1}{|\sin x|} \end{aligned}$$

Conclusion,

$$\boxed{\sqrt{(\cot^2 x + 1)} = \frac{1}{|\sin x|}}$$

It remains to understand when

$$|\sin x| = \sin x$$

As we know

$$\sin x \geq 0, \quad \forall x \in [2\pi n; \pi + 2\pi n], \quad n \in \mathbb{Z}$$

And on the segment $0 \leq x < 2\pi$

$$\sin x \geq 0, \quad 0 \leq x \leq \pi$$

Then,

$$|\sin x| = \sin x, \quad 0 \leq x \leq \pi \rightarrow \boxed{\csc x = \sqrt{(\cot^2 x + 1)}, \quad 0 < x < \pi}$$

Expelled two values $x = 0$ and $x = \pi$ because they $\sin x = 0$ which cannot be, since $\sin x$ is in the denominator of expression

Conclusion,

$$\csc x = \sqrt{(\cot^2 x + 1)}, \quad 0 < x < \pi$$

ANSWER

$$a. \quad 0 < x < \pi$$

Answer provided by <https://www.AssignmentExpert.com>